Burau Representation from Uq(sl2)

Junior Topology Siminar

Mustafa Hajij

Algebras

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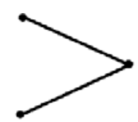
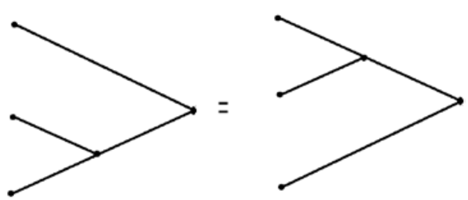
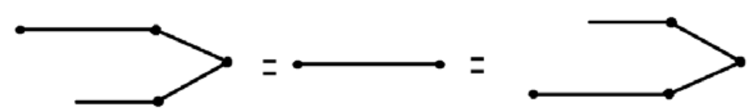
In picture this means there exist two maps  and  such that





Algebras

Let A and A' be k-algebras. There is an k-algebra structure on the tensor product $A \otimes A'$. Multiplication is given by $(m \otimes m')(1 \otimes \tau_{A \otimes A'} \otimes 1)$ and the unit is given by the composition of $u \otimes u'$ with the canonical isomorphism $k \equiv k \otimes k$.

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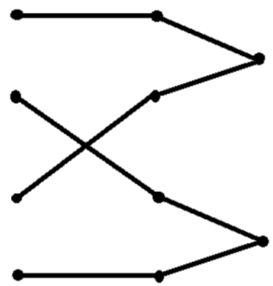

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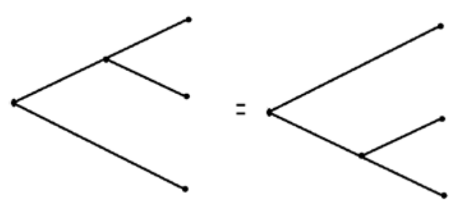

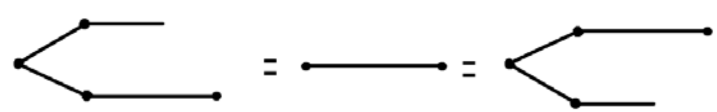

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Let C and C' be coalgebras. A coalgebra structure can be put on the tensor product of two coalgebras. Namely, we define a comultiplication on $C \otimes C$ by $(1 \otimes \tau_{A \otimes A'} \otimes 1)(\Delta \otimes \Delta')$. The counit is given as the composition of the canonical isomorphism $k \otimes k \equiv k$ with $\varepsilon \otimes \varepsilon'$.

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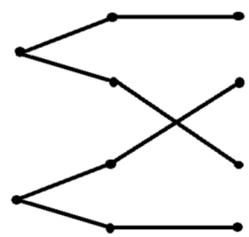

Bialgebras

A k-module A having both algebra and coalgebra structures is called a k-bialgebra if these structures are compatible with each other in the sense that the linear maps $\Delta : C \to C \otimes C$ and $\varepsilon : C \to k$ are in fact morphisms of algebras. Or, what is equivalent, the linear maps $m : A \otimes A \to A$ and $u : k \to A$ are morphisms of coalgebras.

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Thus we have the following lemma.

Lemma 1 *Let A be a bialgebra.The category of A-modules is a monoidal category.*

Braided Categories

A braiding in a monoidal category C consists of family of isomorphisms $c = \{c_{V,W} : V \otimes U \to U \otimes V\}$ where V, W run over all objects of C, such that for any three objects U, V, W, we have

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and for any morphisms $f : V \to V'$ 0 and $g : W \to W'$, we have

$$(g \otimes f)c_{V,W} = c_{V',W'}(f \otimes g)$$

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where the map P is the permutation given by $P(x \otimes y) = y \otimes x$ and we put $\hat{R}_{12} = \hat{R} \otimes 1$, $\hat{R}_{23} = 1 \otimes \hat{R}$ and $\hat{R}_{13} = \sum \alpha_i \otimes 1 \otimes \beta_i$ putting $\hat{R} = \sum \alpha_i \otimes \beta_i$.

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Theorem *Let A be a bialgebra. The category of A-modules is braided iff A is quasi-triangular bialgebra.*

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Theorem *Let A be a braided bialgebra and let c be a braiding in A-Mod. Then for all A-modules U, V and W we have*

$$(c_{V,W} \otimes id_U)(id_V \otimes (c_{U,W})((c_{U,V} \otimes id_W) = (id_W \otimes c_{U,V})((c_{U,W} \otimes id_V)(id_U \otimes (c_{V,W})$$

Braid group representaions..

Suppose that A is any braided bialgebra with braiding c . Suppose further that V is any A-module. Define $c_i = 1 \otimes ... \otimes c_{V,V} \otimes ... \otimes 1$, an automorphism of the n-fold tensor product $V^{\otimes n}$, where the $c_{V,V}$ term occupies the ith and $(i+1)$st places. Using this we obtain a representation $B_n \to GL(V^{\otimes n})$ given by $\sigma_i \to c_i$.

Burau Representation

Let t be a non-zero complex number and let b_i be the $n-1 \times n-1$ matrices

$$b_1 = \begin{pmatrix} -t & 0 & & & & & \\ -1 & 1 & & & & & 0 \\ & & 1 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & \cdot \\ & & 0 & & & & 1 \end{pmatrix}, \; b_{n-1} = \begin{pmatrix} 1 & & & & & & \\ & 1 & & & & & 0 \\ & & \cdot & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & 1 & -t \\ 0 & & & & & 0 & -t \end{pmatrix}$$

$$b_i = \begin{pmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & -t & 0 & & \\ & & 0 & -t & 0 & & \\ & & 0 & -1 & 1 & & \\ & & & & & 1 & \\ & & & & & & 1 \end{pmatrix}$$

$2 \leq i \leq n-2$ where the diagonal $-t$ is in the $i-i$ position. One might easily check that $b_i b_{i+1} b_i = b_{i+1} b_i b_{i+1}$ and $b_i b_j = b_j b_i, |i-j| \geq 2$

One can send σ_i to b_i and get the so-called reduced Burau representation of B_n.

The bialgebra $U_q(sl2)$

Definition *Let $q \neq 0, 1, -1$ in $\mathbb{C}$. The quantum group $U_q(sl(2))$ is the algebra over $\mathbb{C}$, with the unit element 1, generated by E, F, K and K^{-1}, subject to the relations*

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One could equip $U_q(sl(2))$ with a bialgebra structure. Further, one could check that the element $R = q^{-\frac{H \otimes H}{2}} (\sum_{n=0}^{\infty} q^{\frac{n(n-1)}{2}} \frac{(1-q^2)^n}{[n]_q!} E^n \otimes F^n)$ equips $U_q(sl(2))$ with a quasi-triangular bialgebra structure.

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$$\begin{aligned} Kv_l &= q^{\lambda-2l}v_l \\ Fv_l &= v_{l+1} \\ Ev_l &= [l]_q[\lambda+1-l]_q v_{l-1} \end{aligned}$$

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$$(\rho\sigma_i)\hat{u}_j = q^{-\frac{1}{2}\lambda^2}\hat{u}_j \qquad \text{for } j \not\mid i, i+1$$

$$(\rho\sigma_i)\hat{u}_{i+1} = q^{-\frac{1}{2}\lambda(\lambda-2)}\hat{u}_i$$

$$(\rho\sigma_i)\hat{u}_i = q^{-\frac{1}{2}\lambda(\lambda-2)}\left[\hat{u}_{i+1} + (q^{-\lambda} - q^{\lambda})\hat{u}_i\right]$$

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So the representation $B_n \to GL((V(\lambda)^{\otimes n})$ restricts to $B_n \to GL((V(\lambda)^{\otimes n})_1)$

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Setting $t = q^{2\lambda}$ we have the reduced Burau representation

Ref.

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Thank You