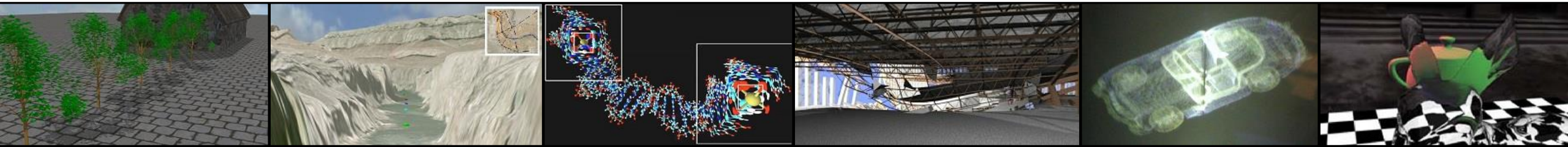
The Shape of an Image: A Study of Mapper on Images

Alejandro Robles
Mustafa Hajij
Paul Rosen

University of South Florida



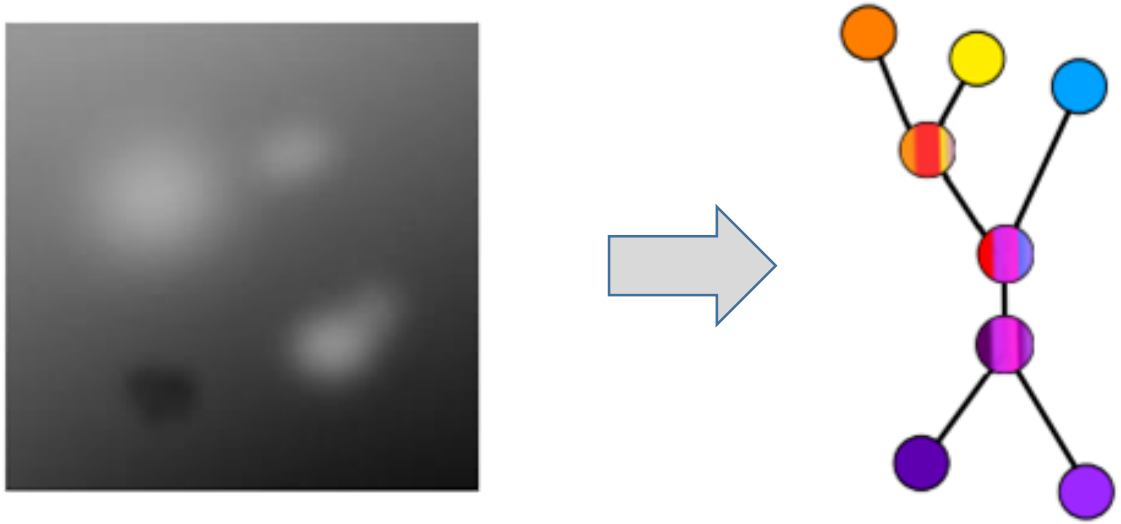
Motivation

Scalar function is defined on planar domain

We would like to extract a topological fingerprint

Why?

Topological fingerprint as a feature vector is translation, rotation, and resolution invariant

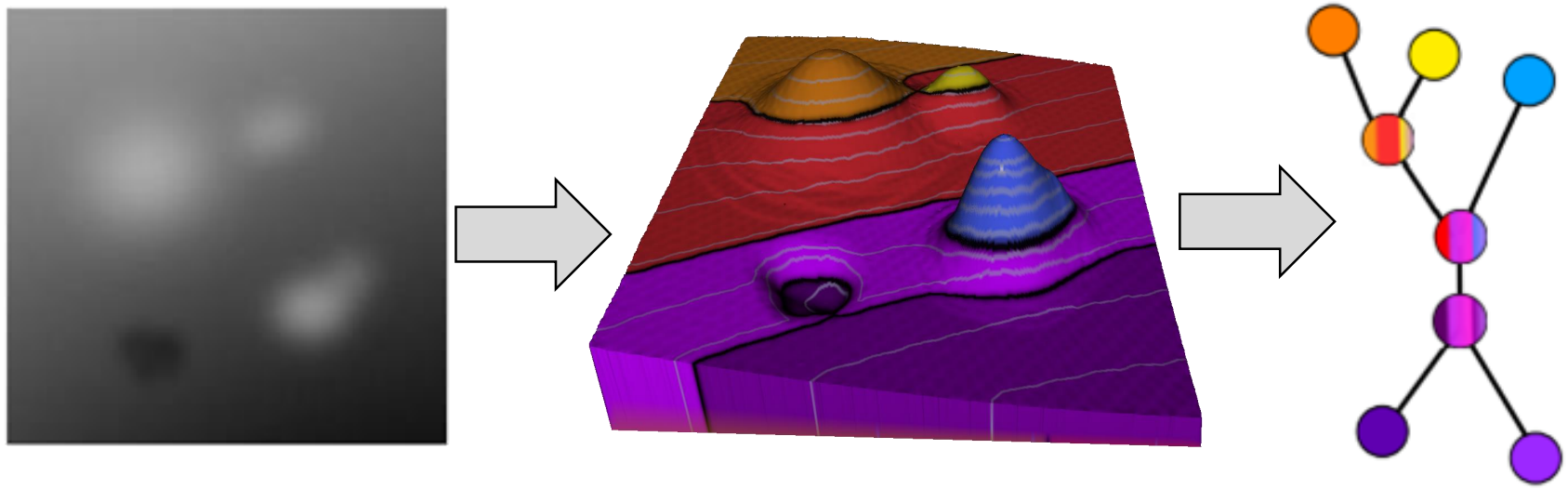


Process Overview

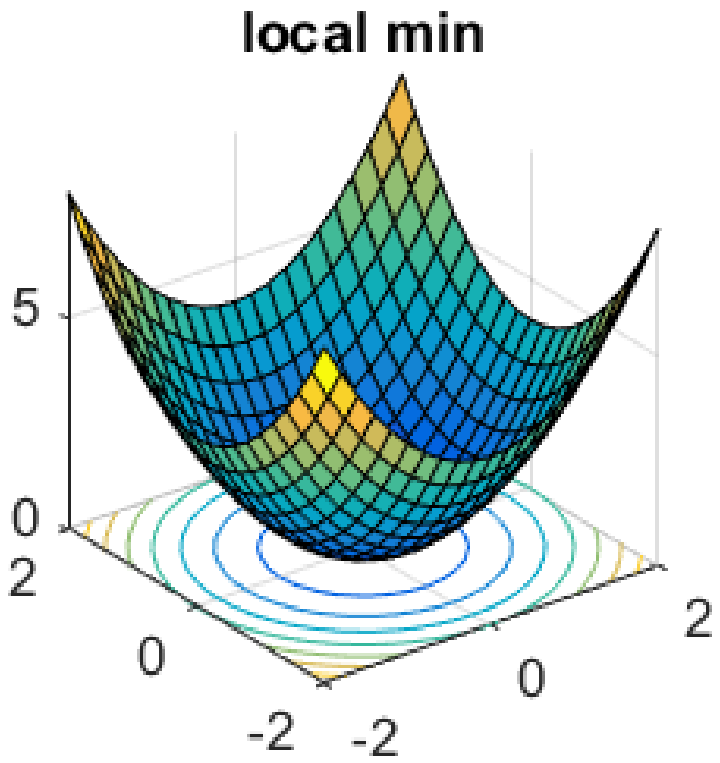
Convert the scalar field into a landscape

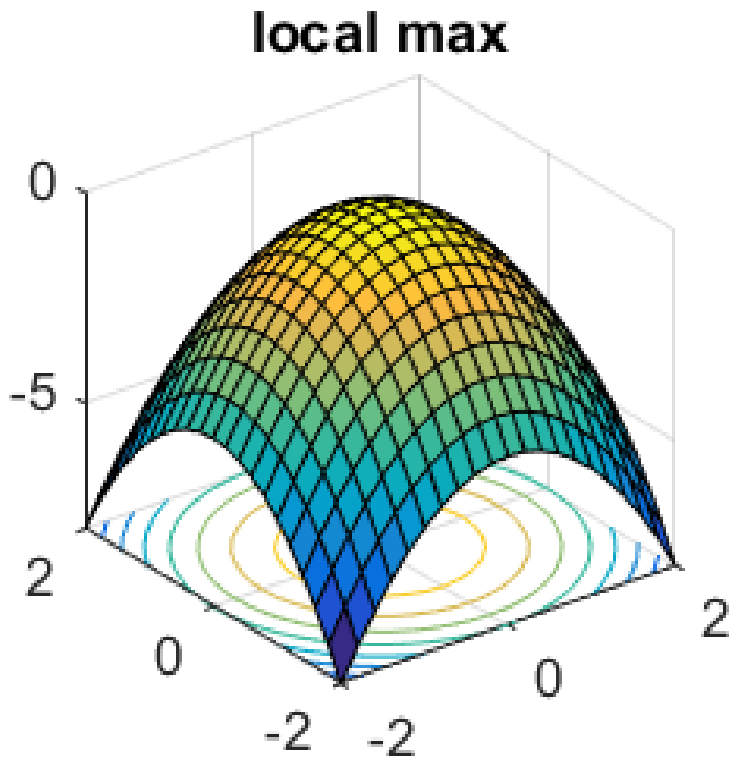
Segment the domain into topological regions by value

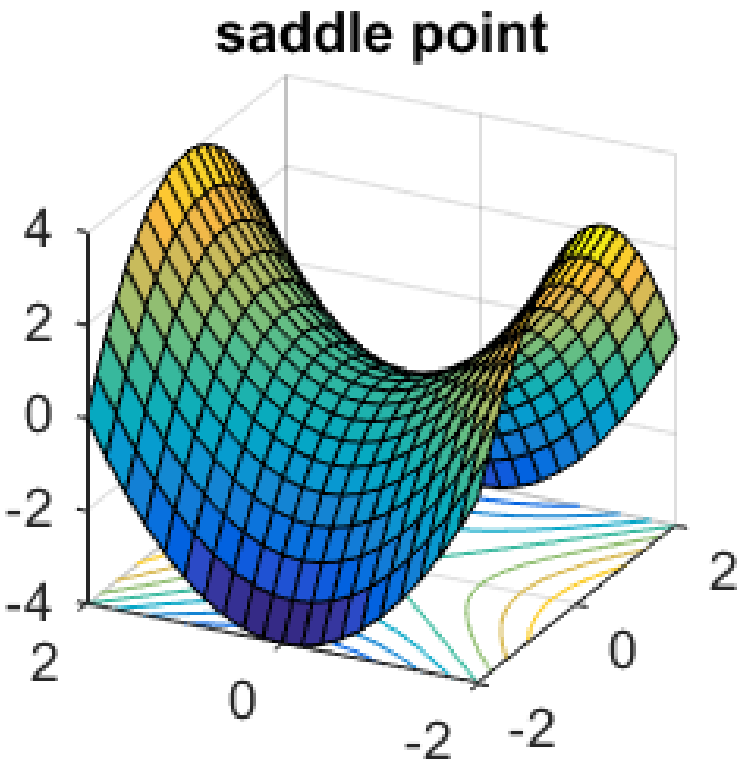
Use the relationship between those regions to describe the topology of the domain



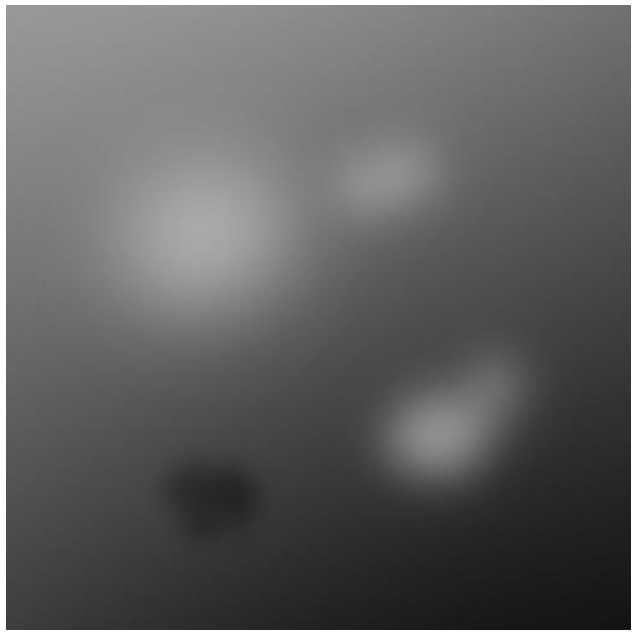
Topological Features

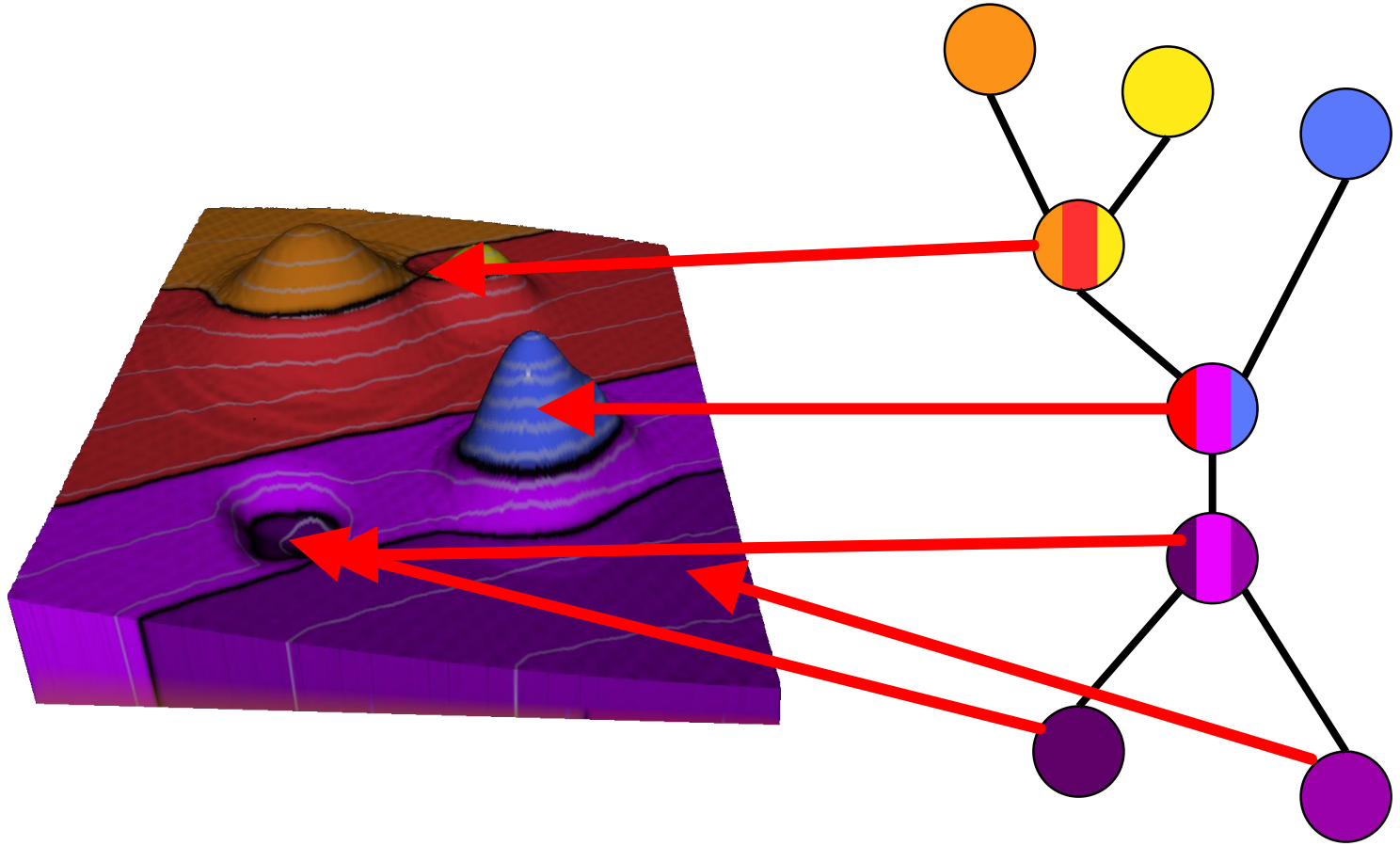






Topology of the Field (Contour Tree)





MAPPER

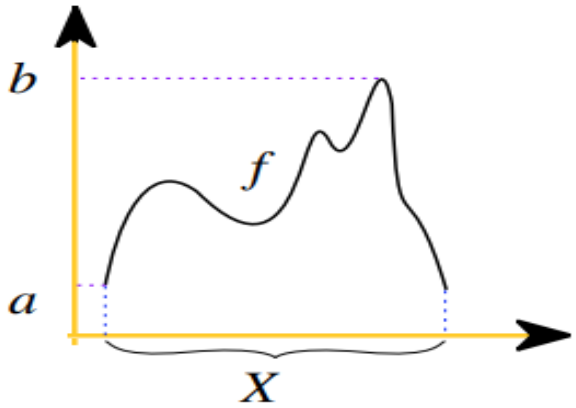
Computation of approximate topology

More flexible and robust implementation & runs faster than contour tree

Ability to control approximation resolution through modifying the “cover”

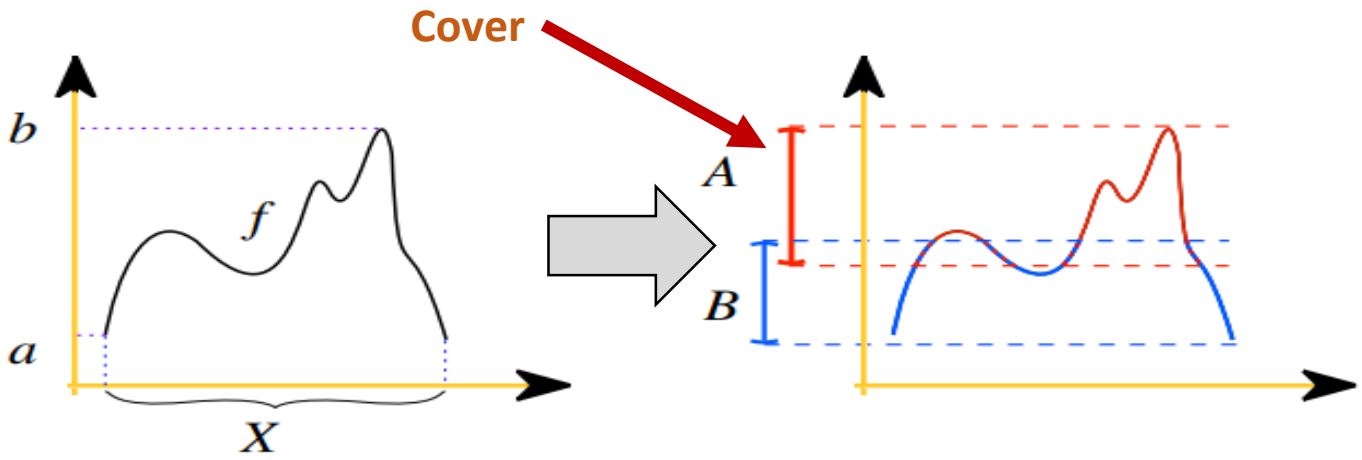
The construction of Mapper on a 1D function

Consider a scalar function $f : X \rightarrow [a,b]$



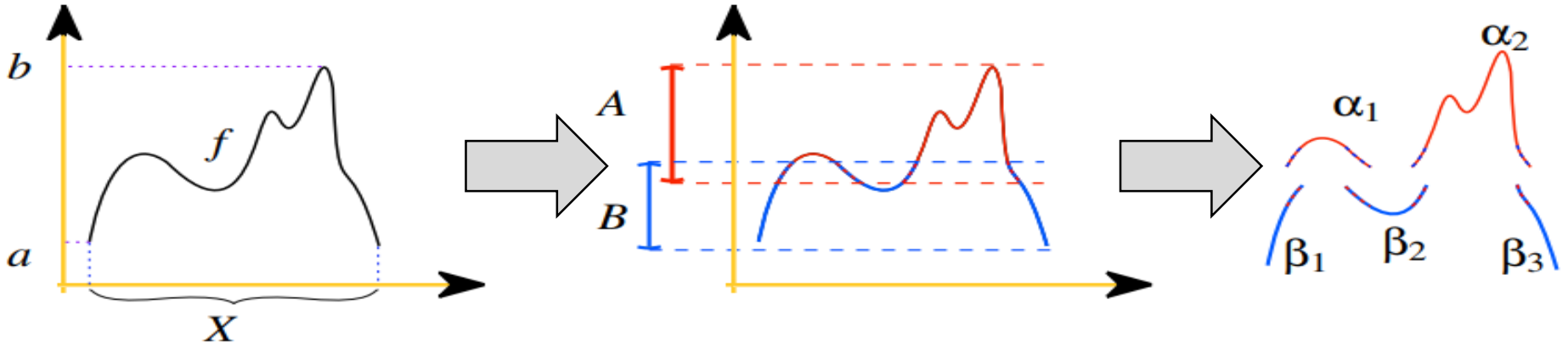
The construction of Mapper on a 1D function

The range [a,b] is covered by the two intervals A,B



The construction of Mapper on a 1D function

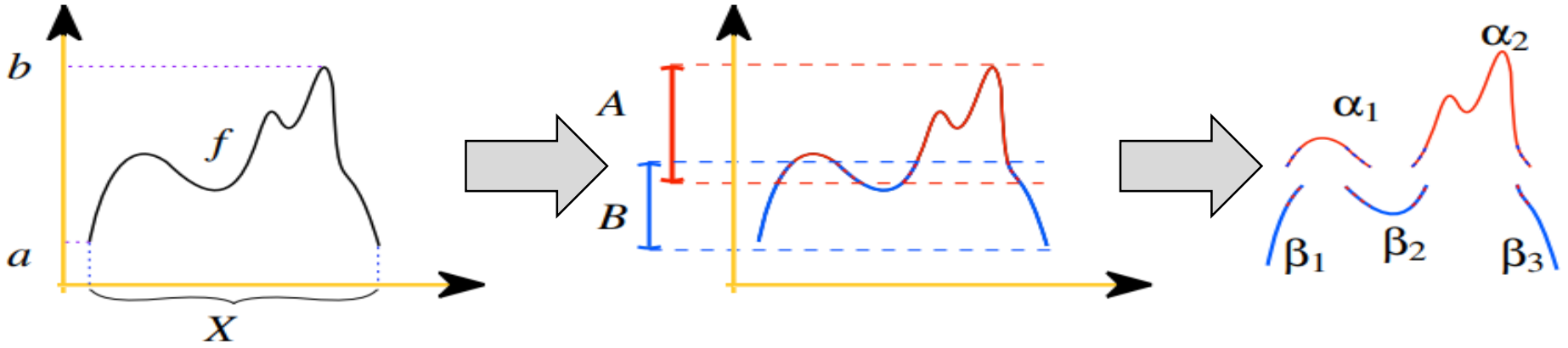
This gives a decomposition of the domain the domain X



The construction of Mapper on a 1D function

Inverse of A consists of 2 connected components α_1 and α_2

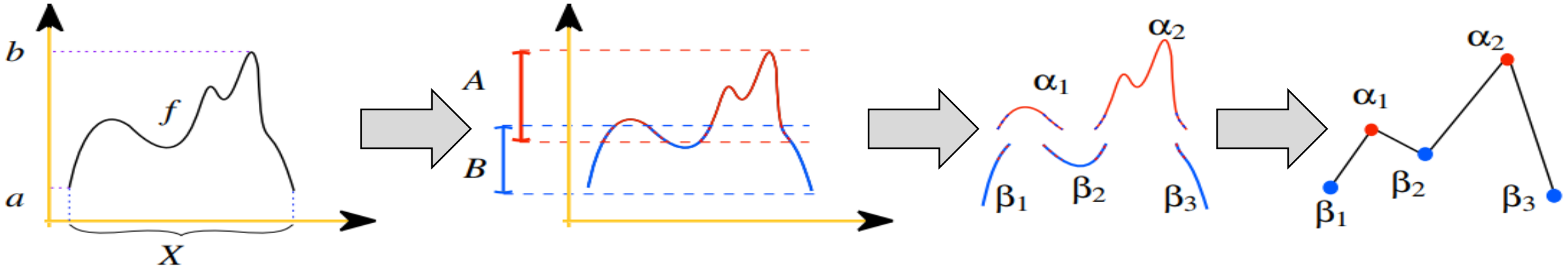
Inverse of B consists of 3 connected components β_1, β_2 and β_3



The construction of Mapper on a 1D function

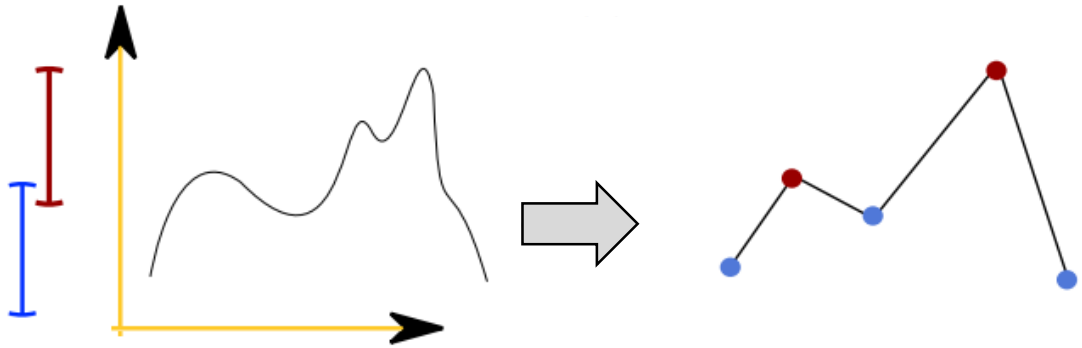
A ***nodes*** represents each connected component

An ***edge*** is inserted whenever 2 connected components overlap

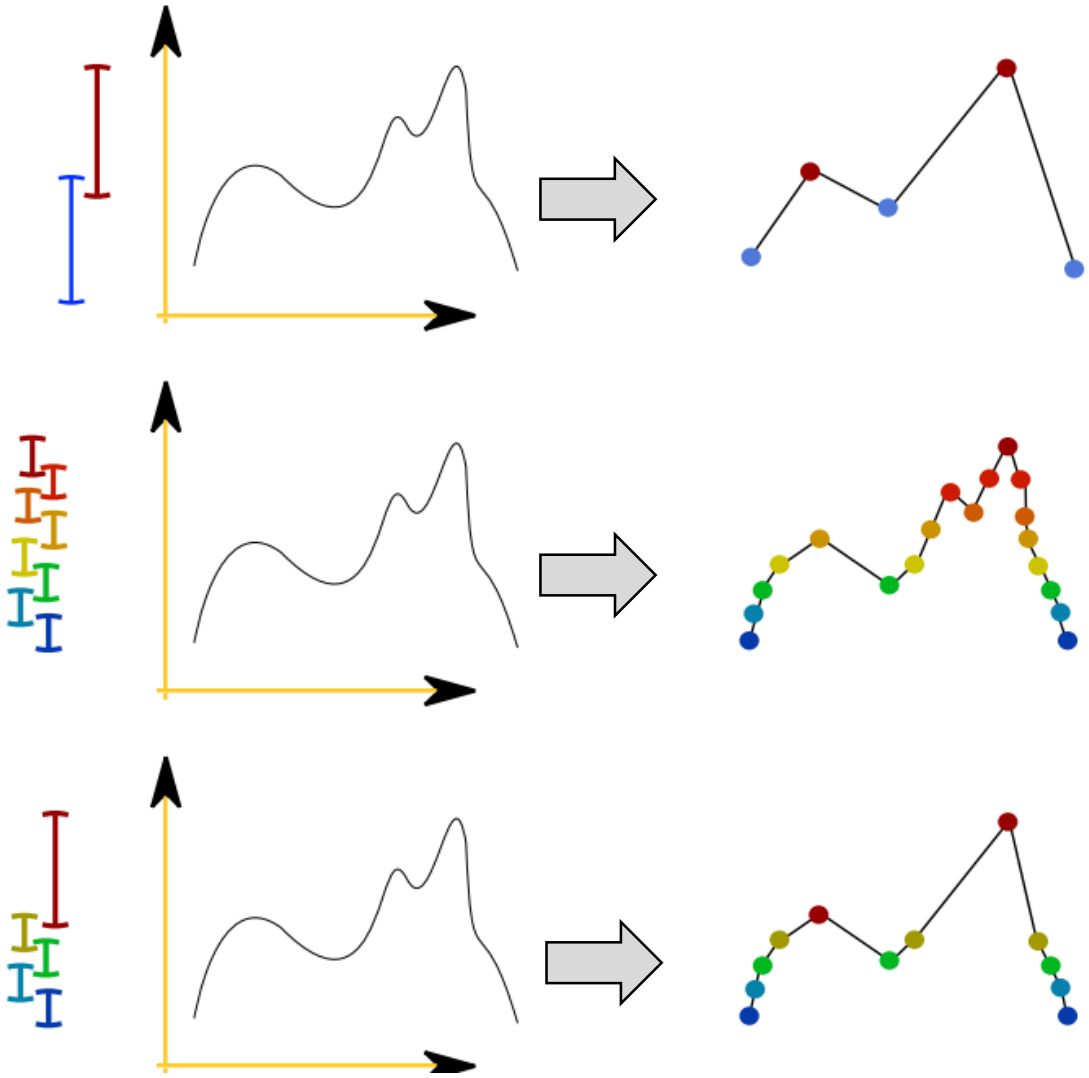


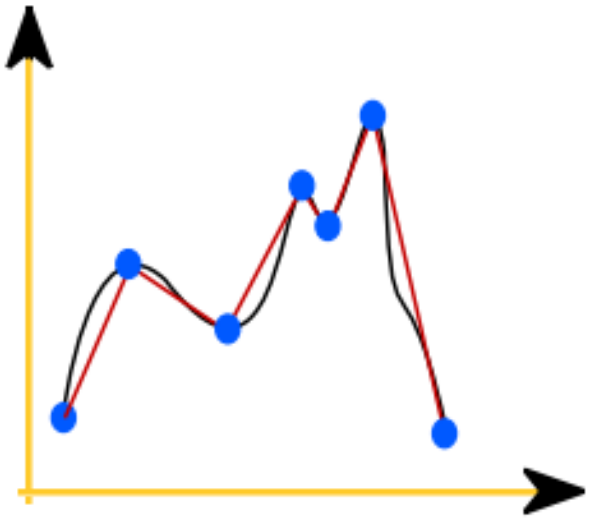
MAPPER RESOLUTION

The construction of mapper depends on the cover chosen for the range [a,b] of the scalar function.



Comparison to Contour Tree

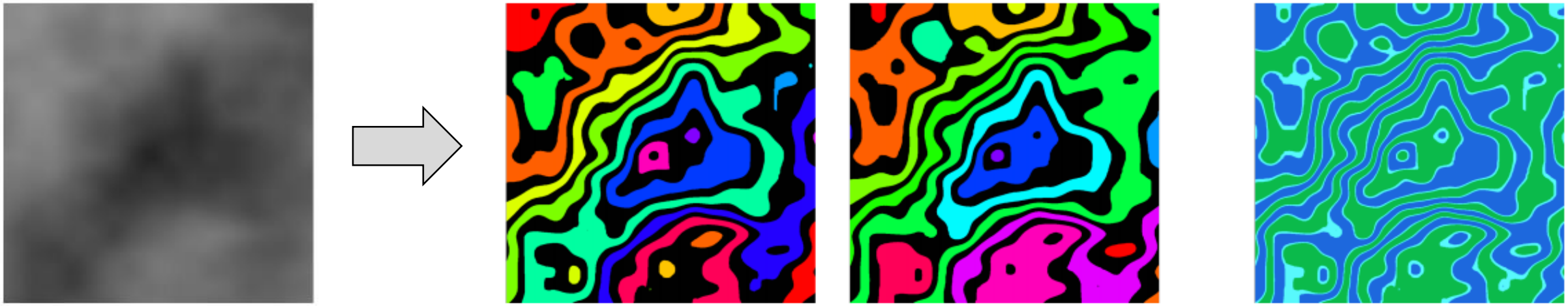




The contour tree of a 1d function.

Computation of Mapper on images

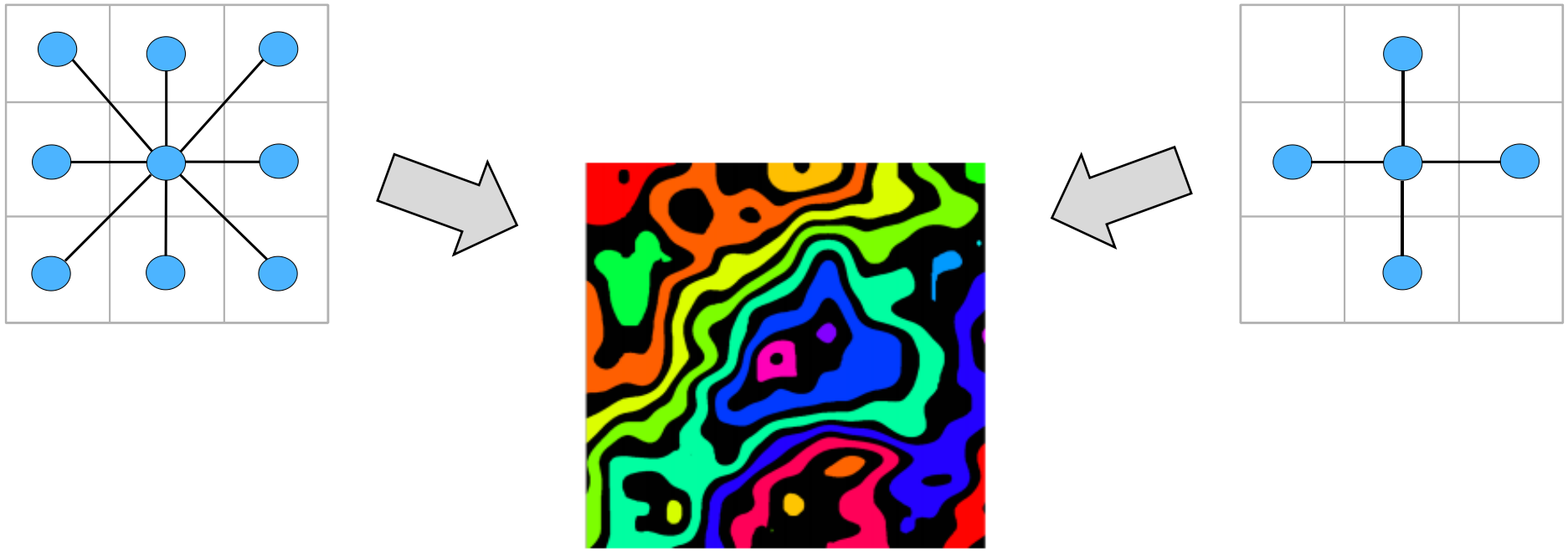
Covers are first extracted as even and odd covers, plus overlap used later.



Input Even Odd Overlap

FINDING VERTICES

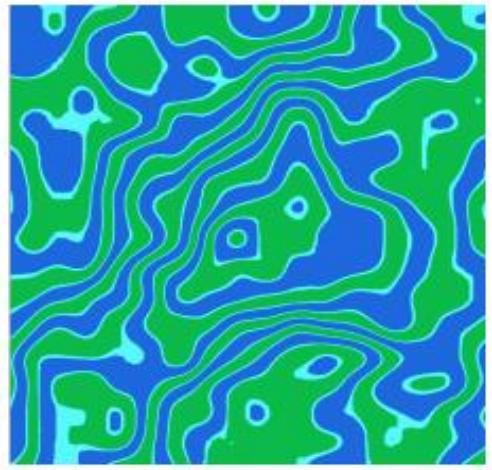
Covers are converted to graphs using 1 of 2 schemes, and connect components identified using either DFS or BFS



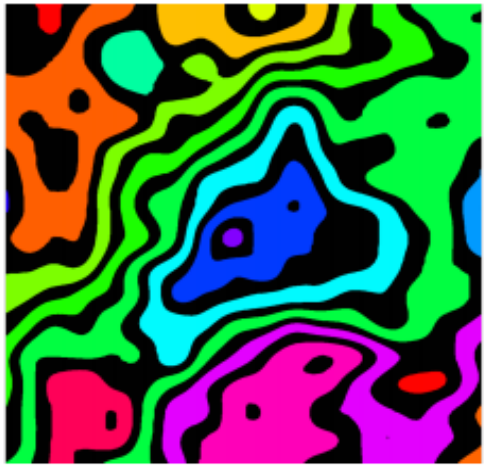
Finding Edges

Pixels from overlapping regions cause edges between connected components in adjacent covers

Even



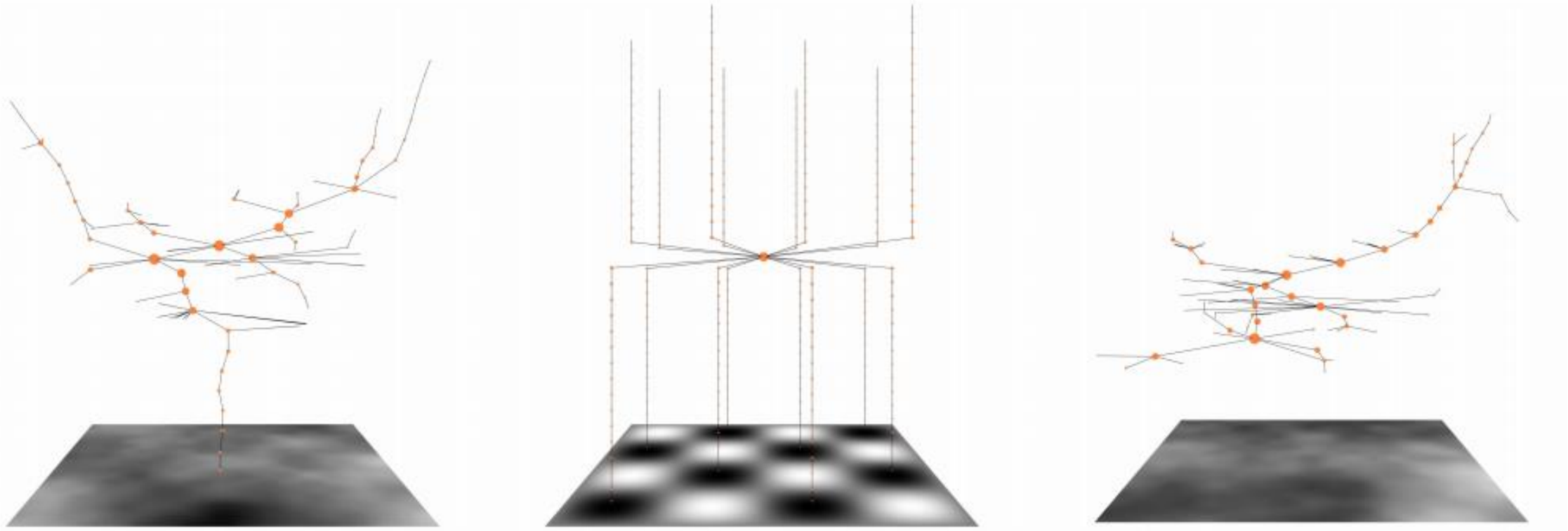
Overlap



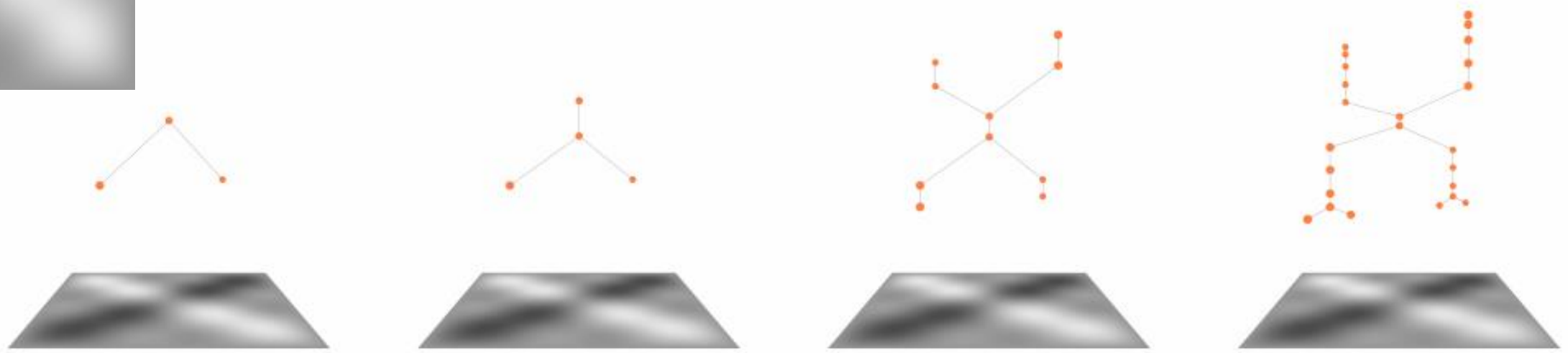
Odd

RESULTS

MAPPER ON IMAGES

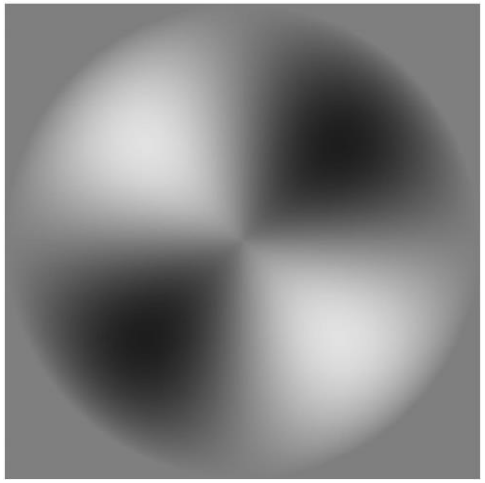


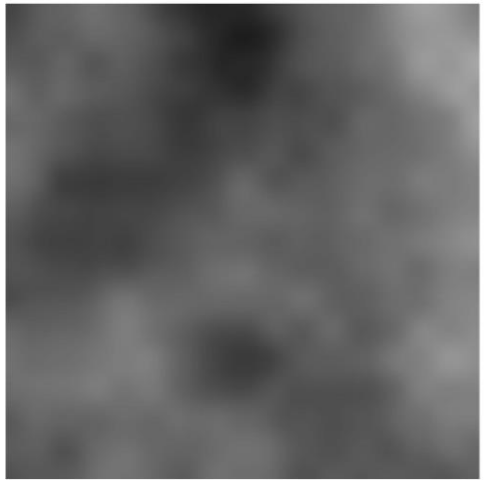
Adjusting Mapper resolution

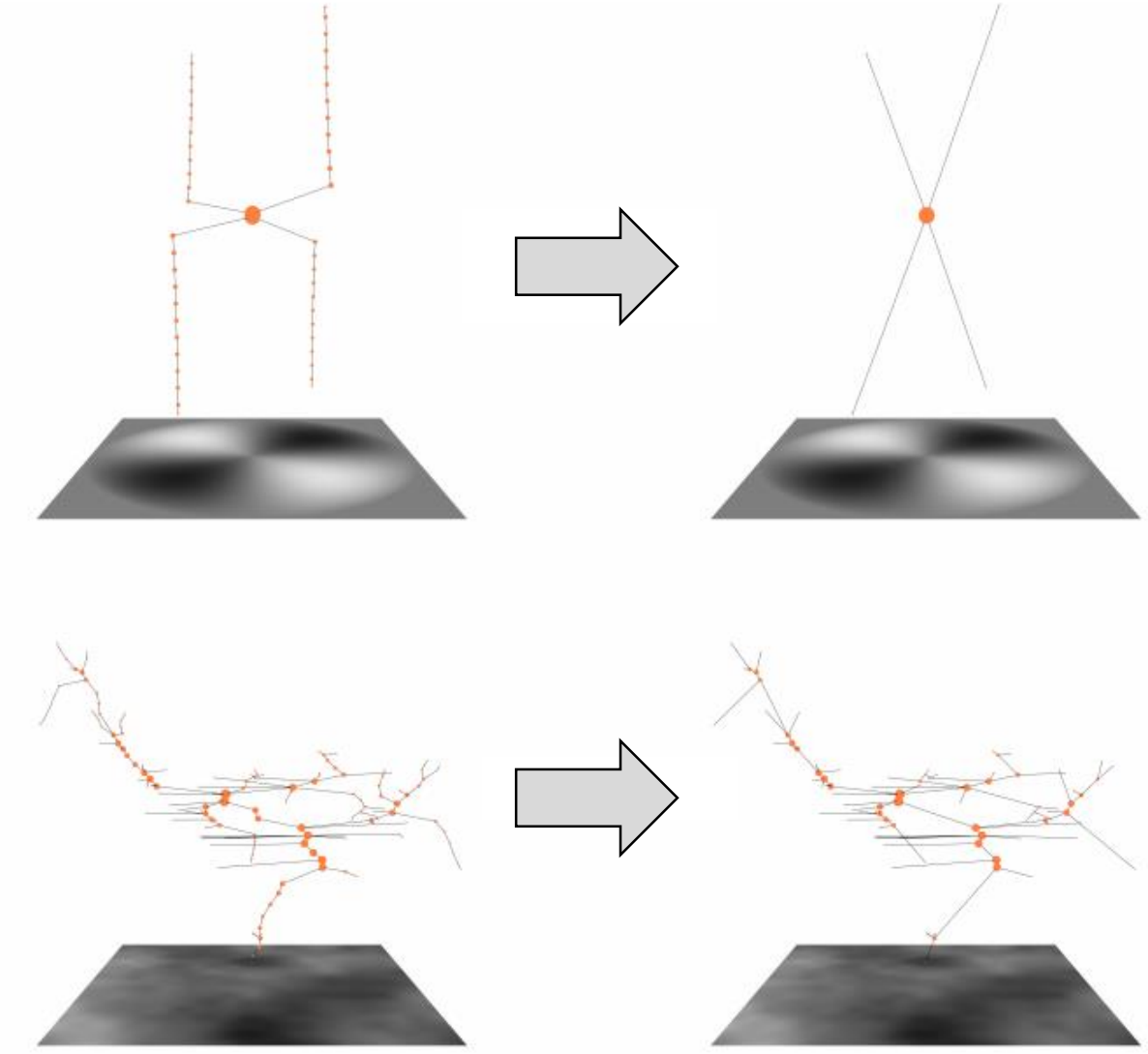


2 cover slices | 4 cover slices | 8 cover slices | 16 cover slices

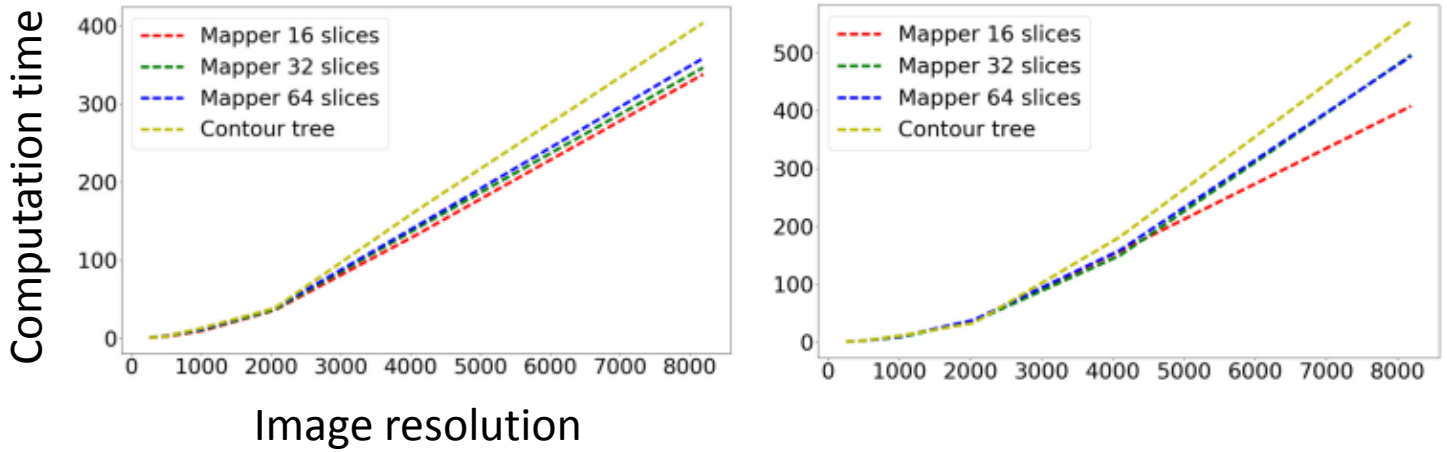
Simplification of Mapper Graphs

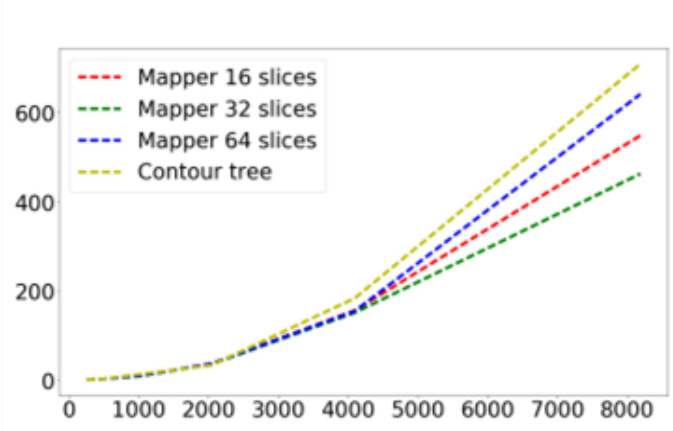


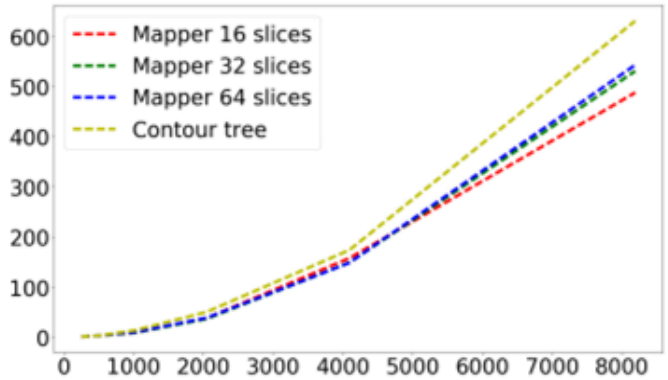




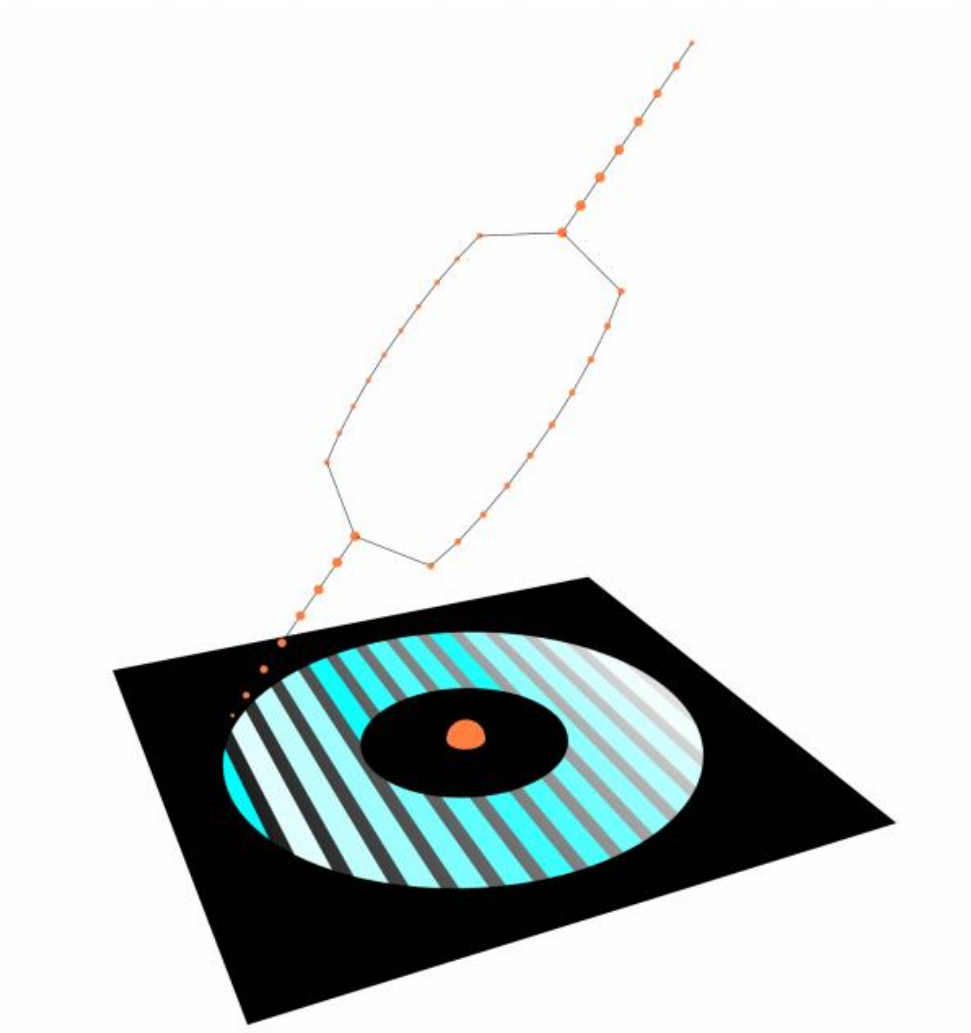
MAPPER PERFORMANCE ON 4 DIFFERENT IMAGES

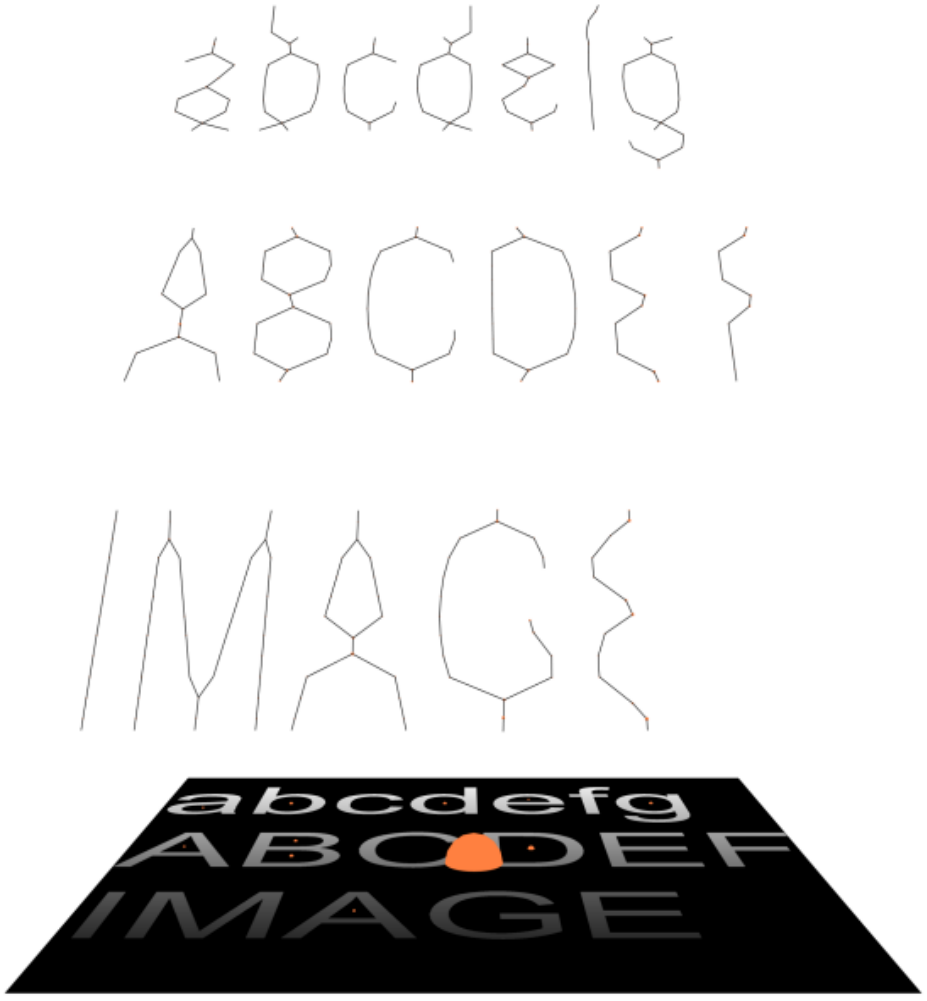






Additional Results





Conclusions

Mapper on image is fast and flexible to compute

Topological fingerprint could serve as a good feature vector, given that it is translation, rotation, and image resolution invariant

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Contact:
Paul Rosen
prosen@usf.edu
http://www.cspaul.com