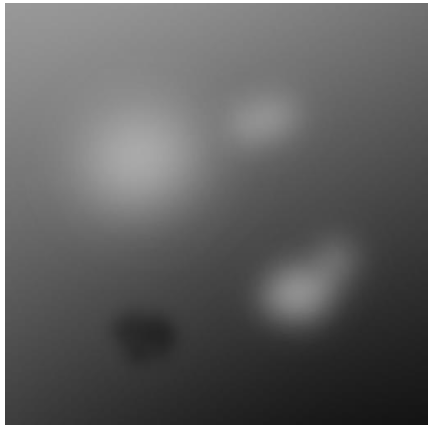
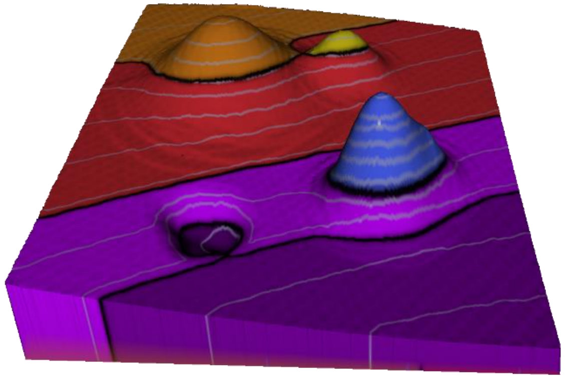
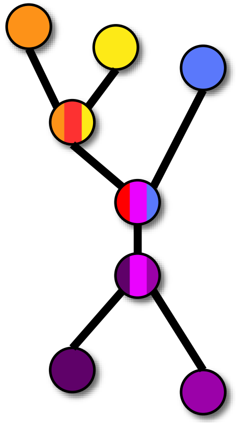
Contour Trees and Persistence







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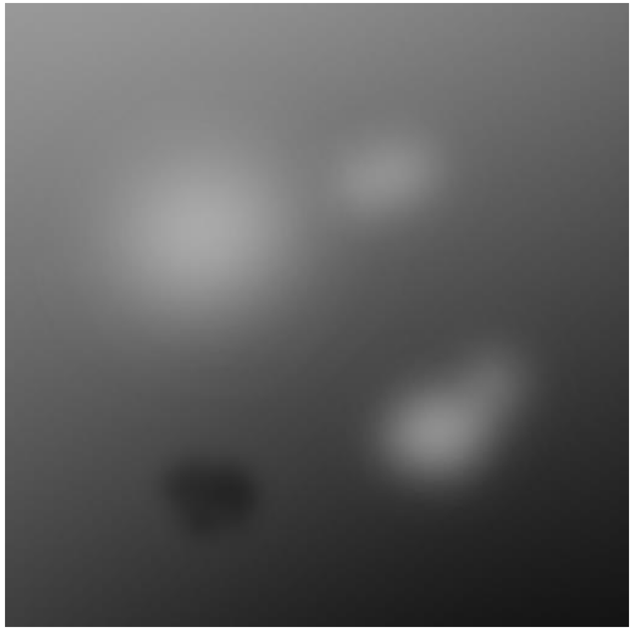
CONTOUR TREES



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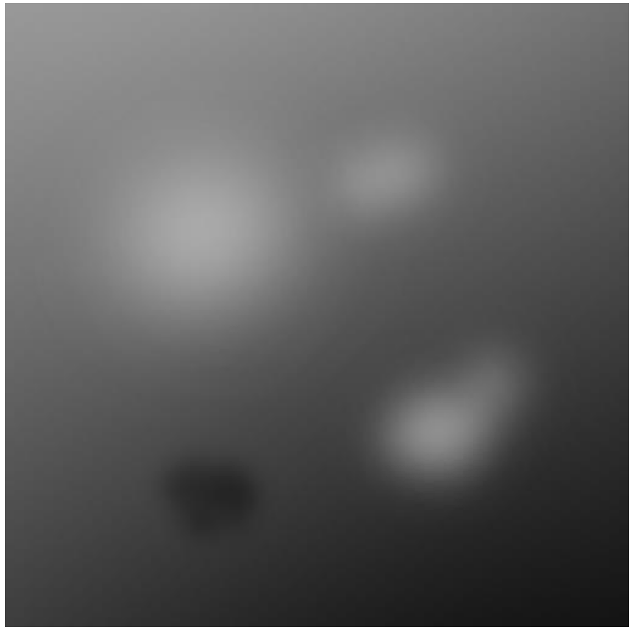

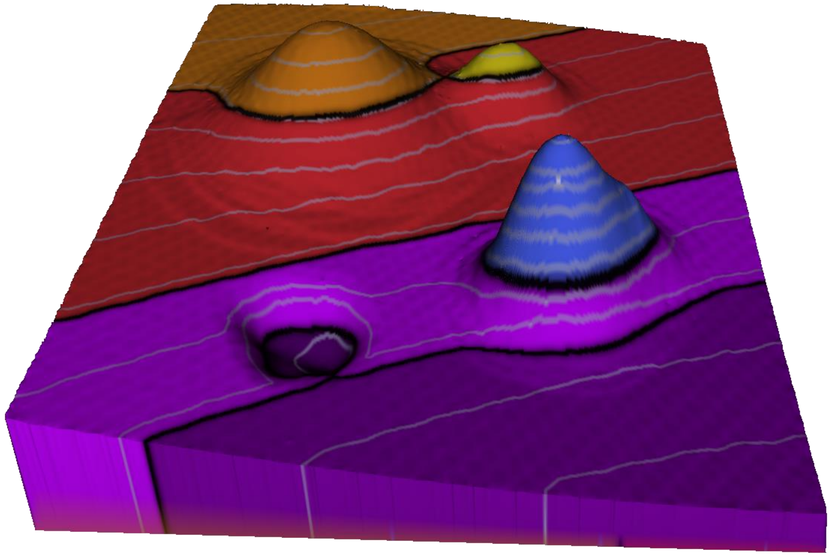

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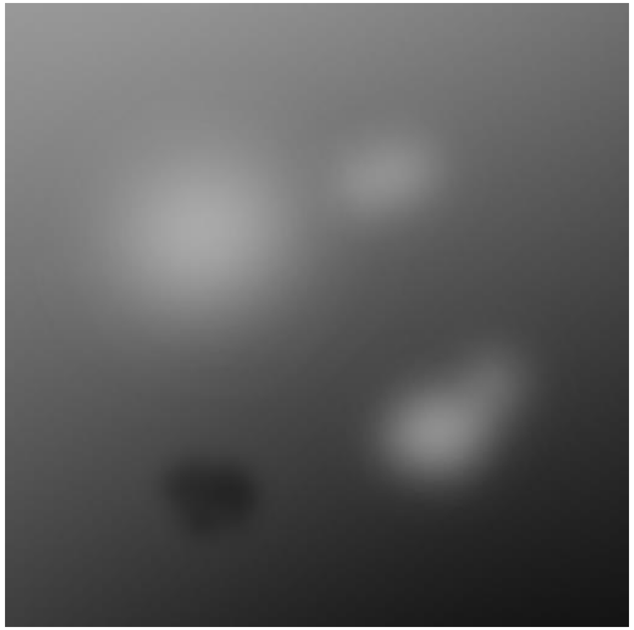
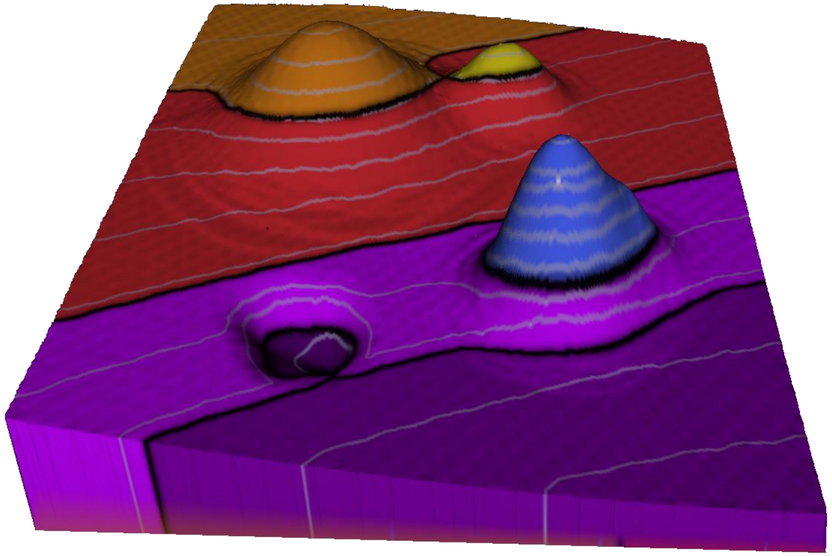
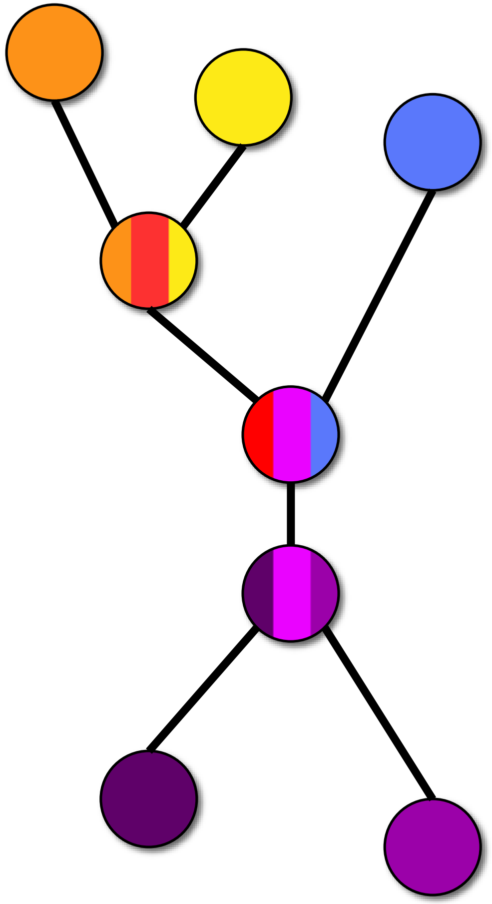
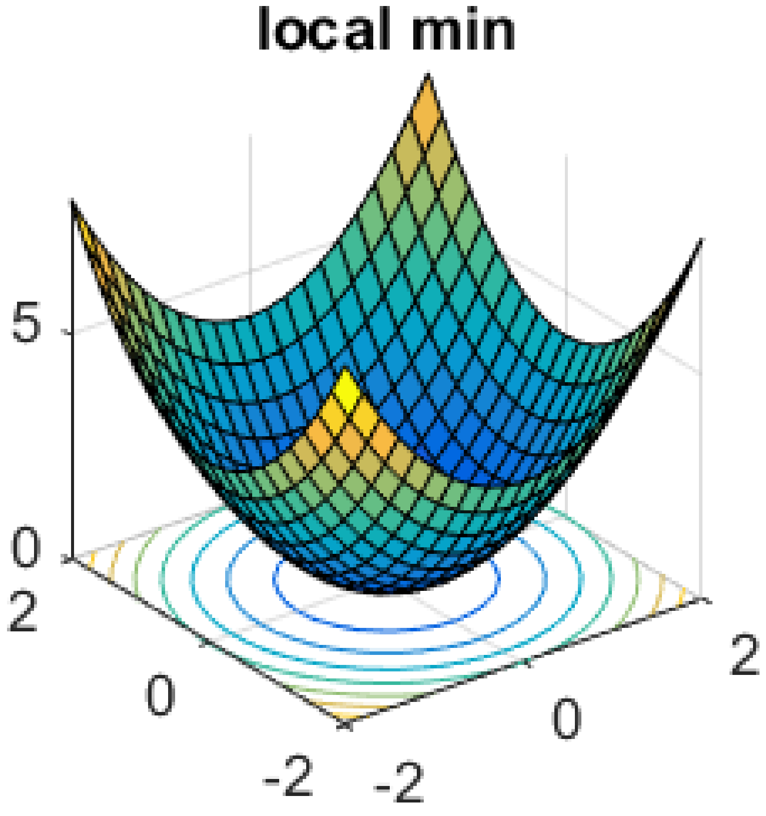
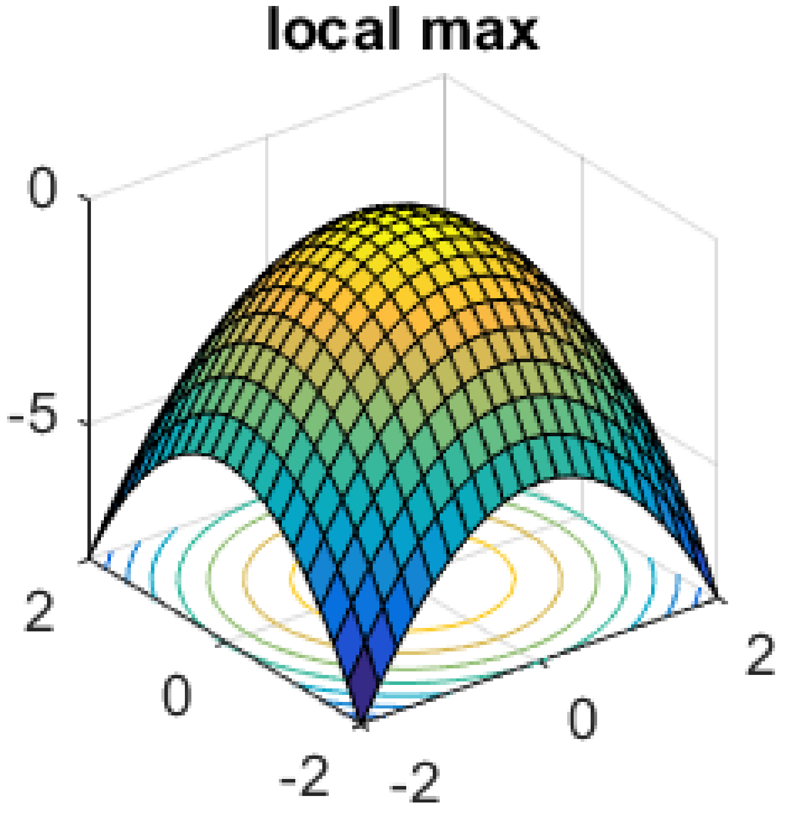


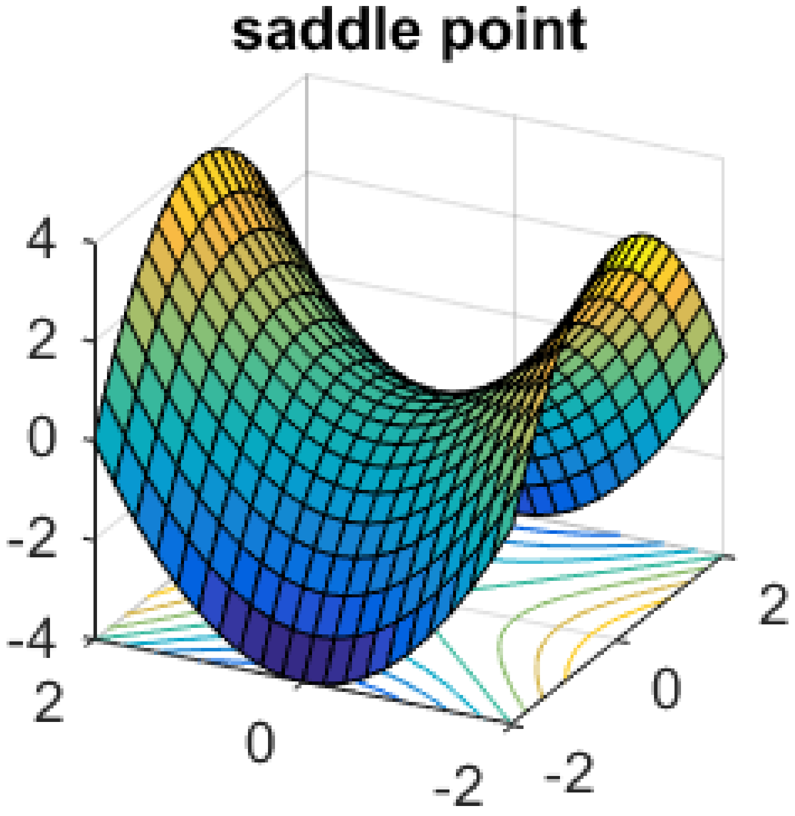




Critical Point Types







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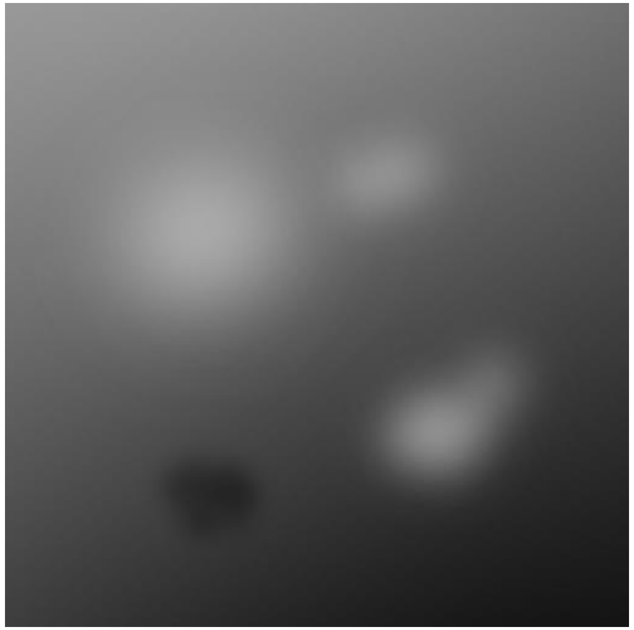

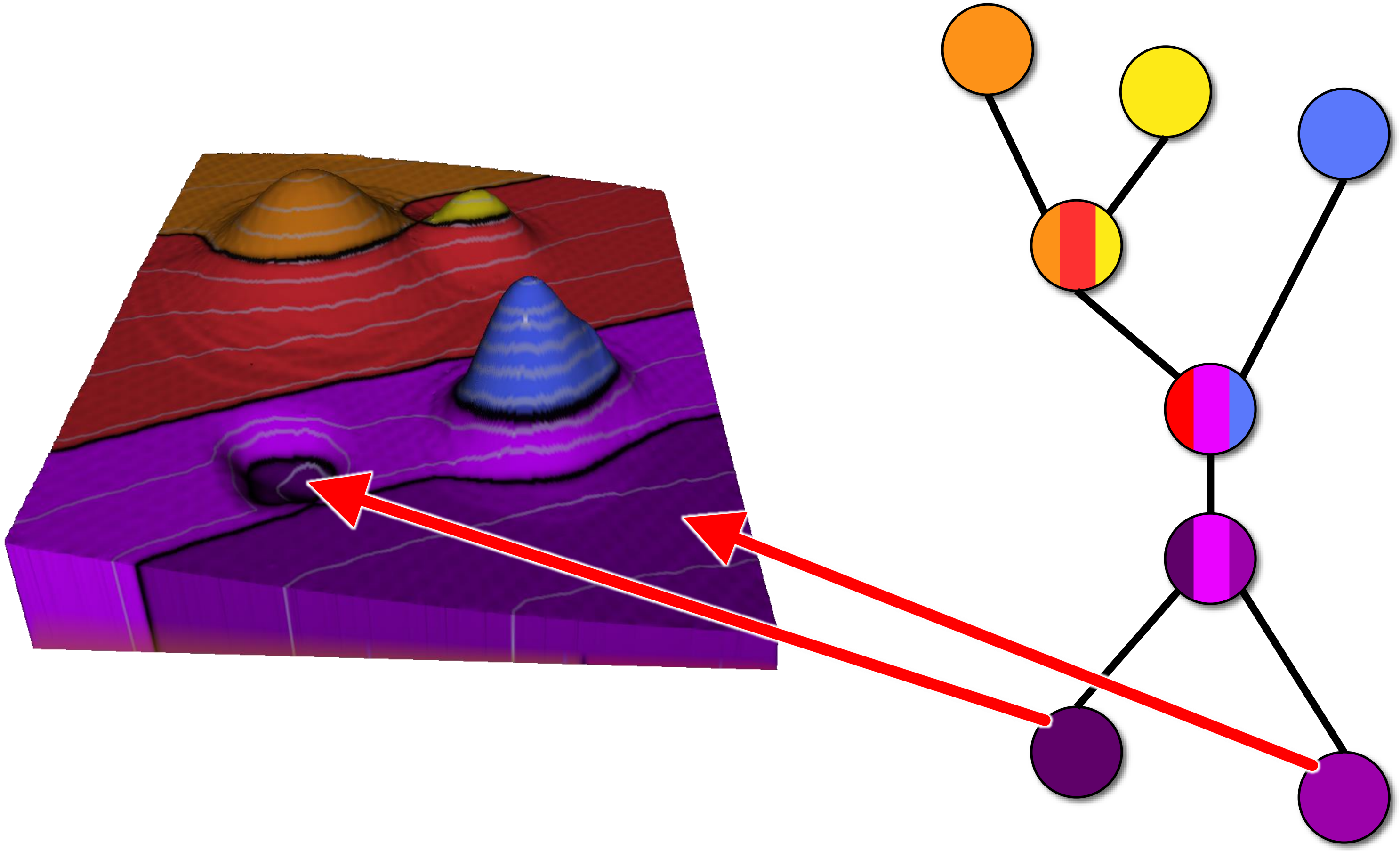

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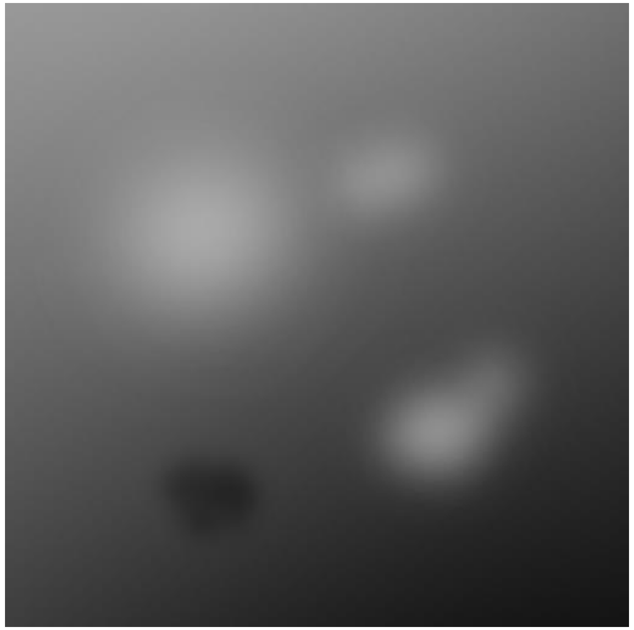

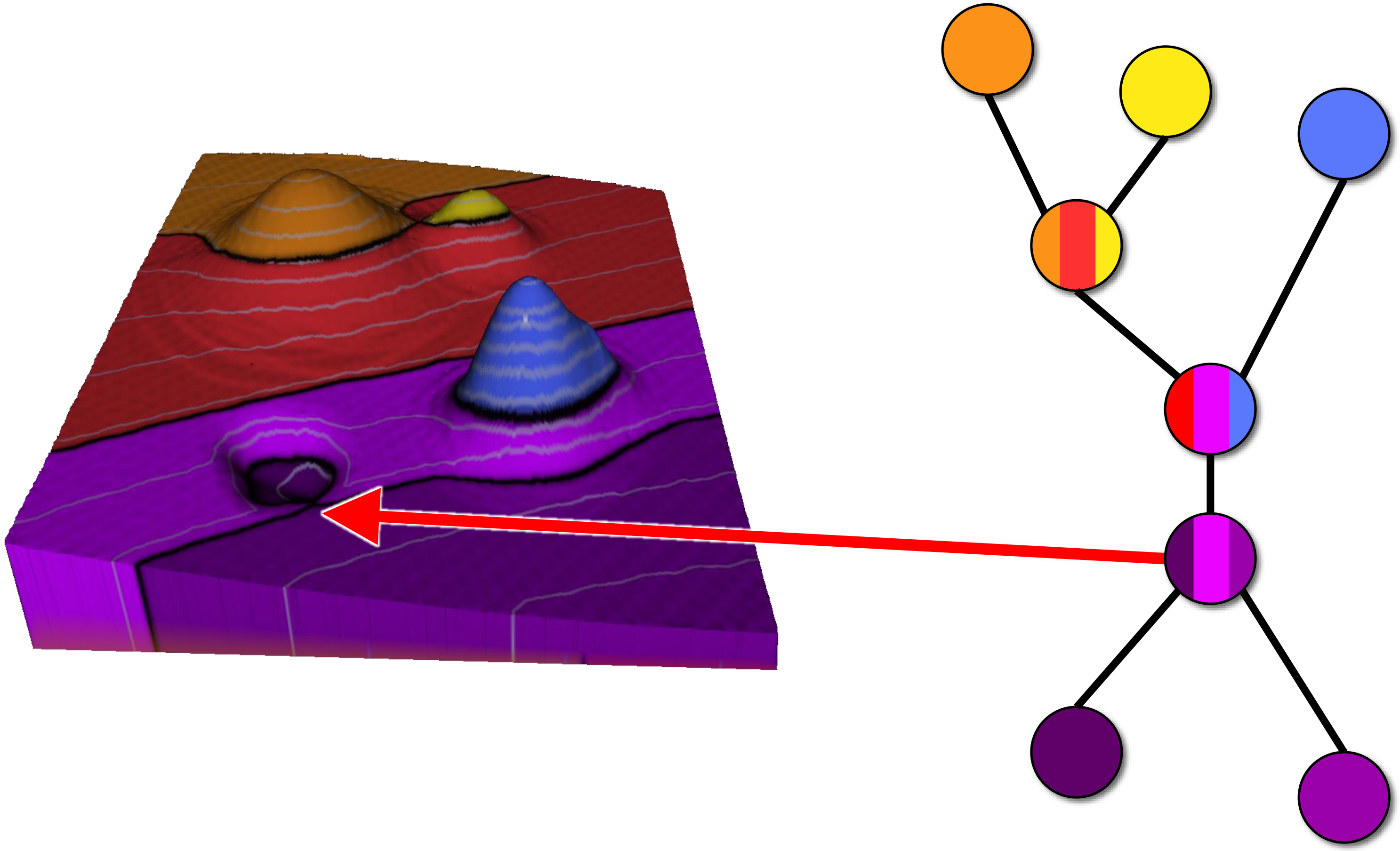

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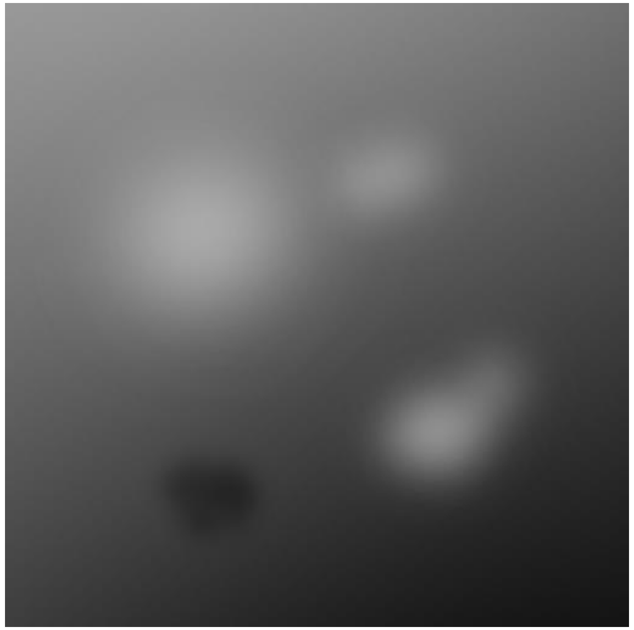
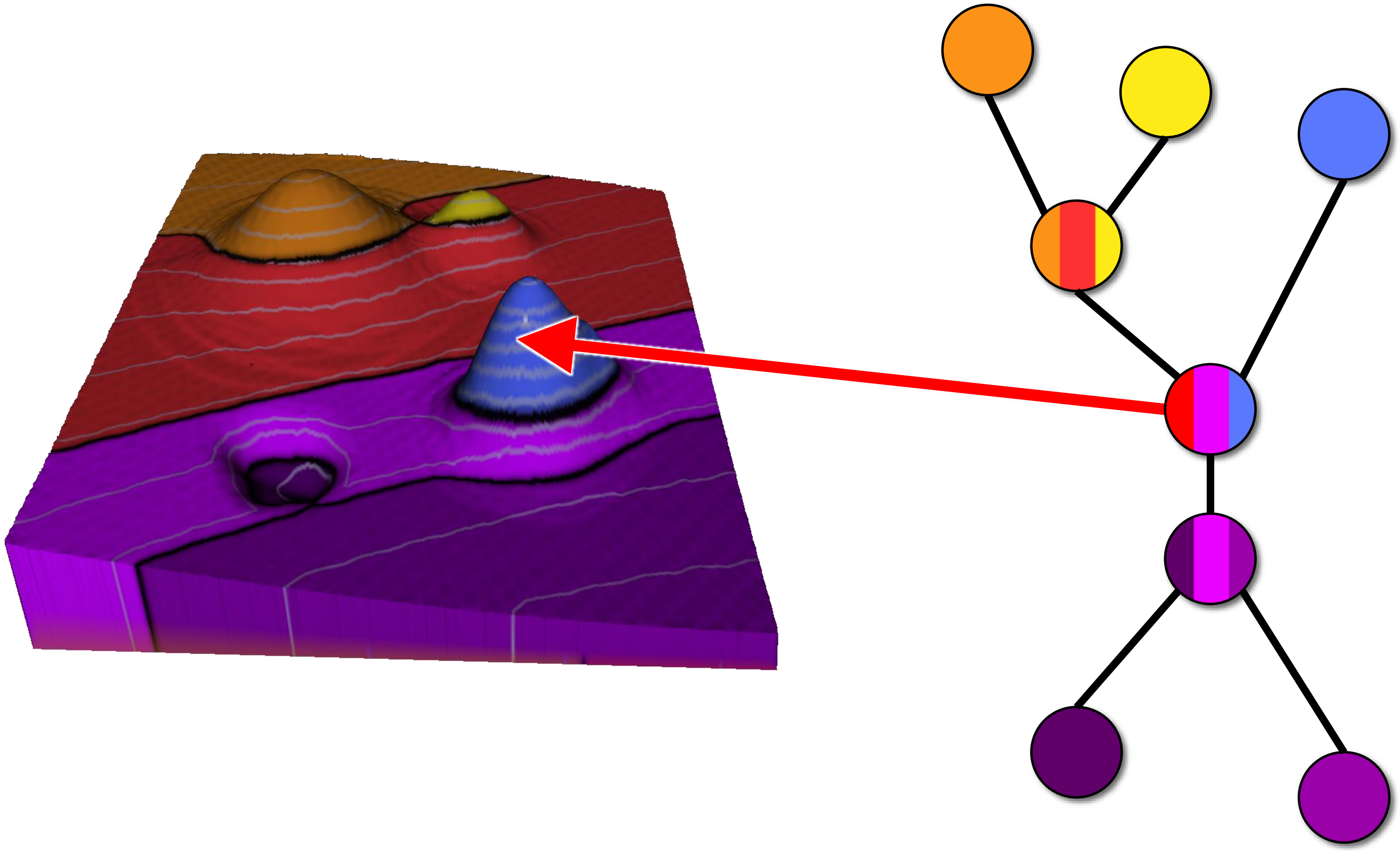
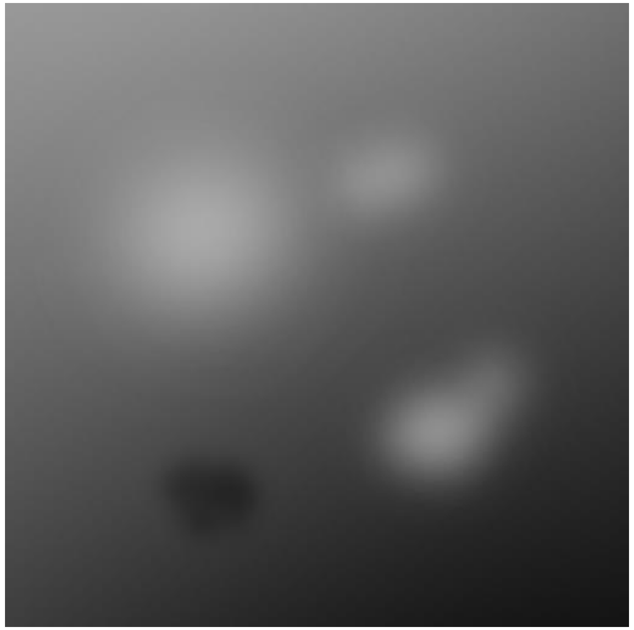
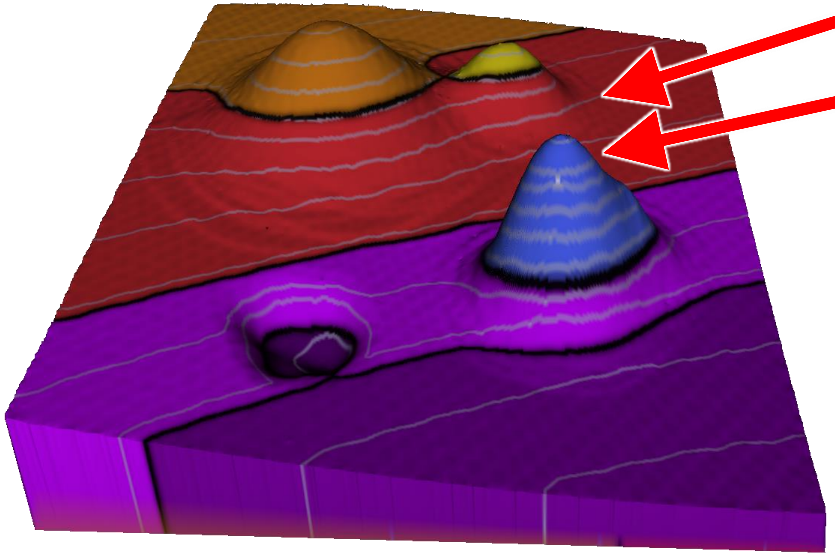
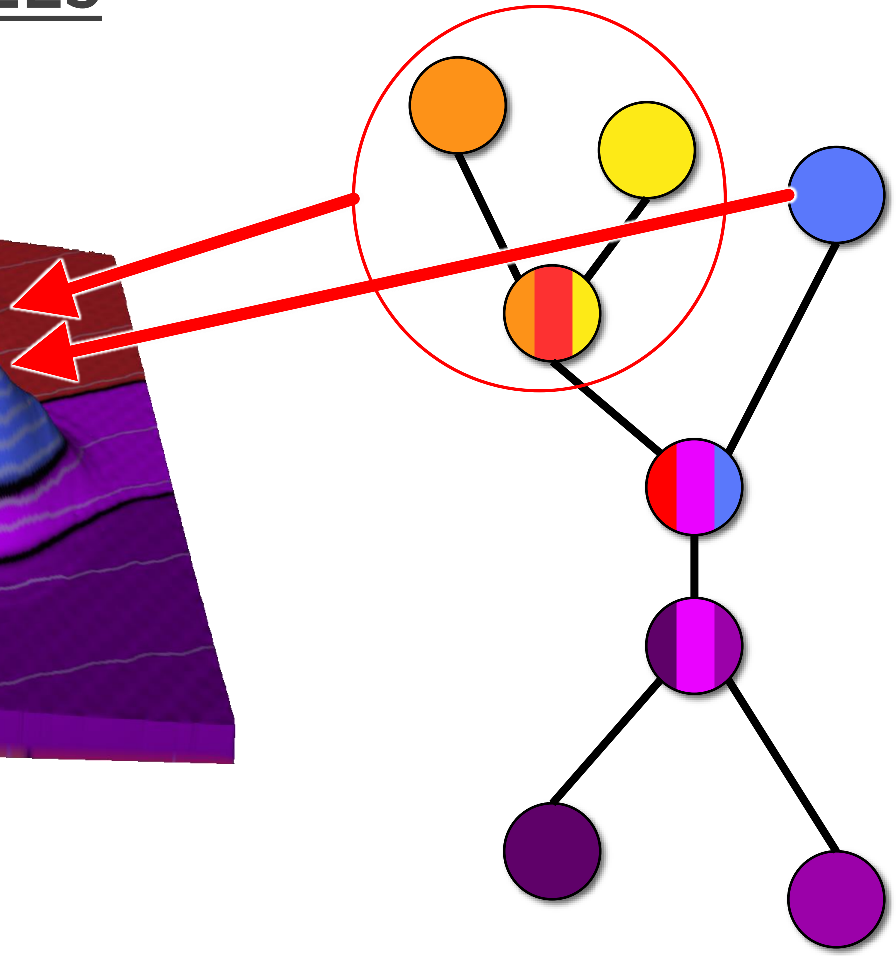




CONTOUR TREES







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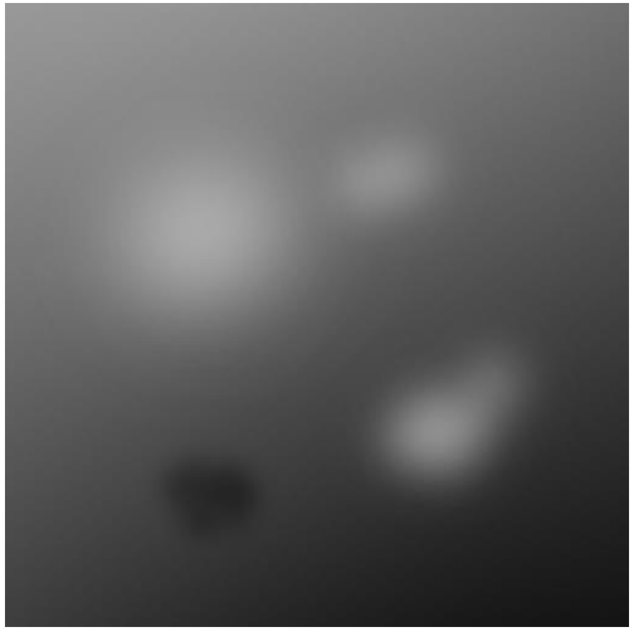
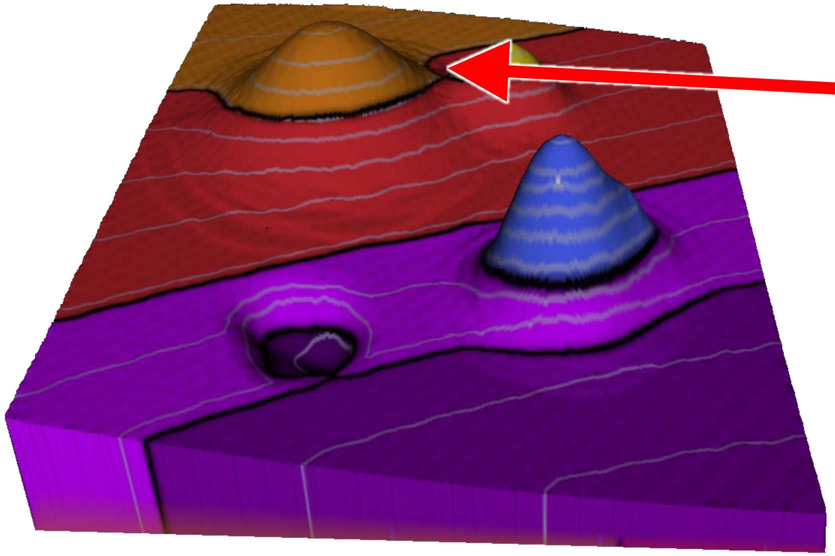
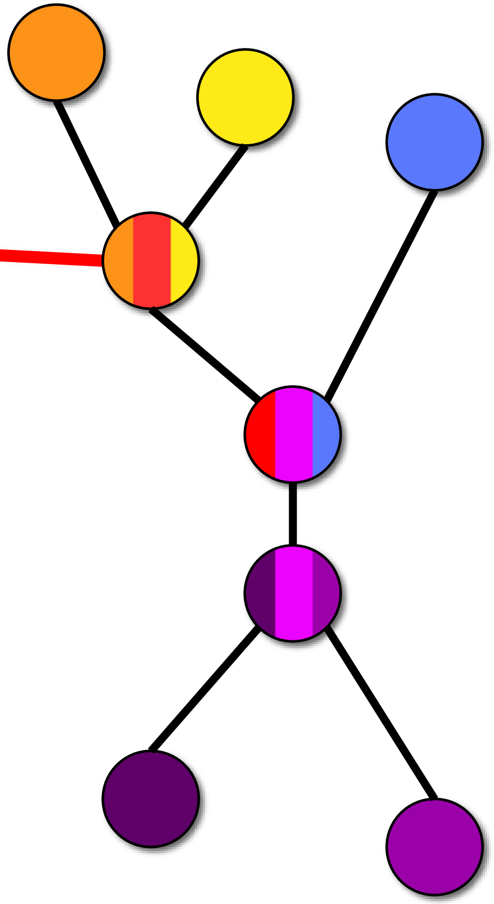
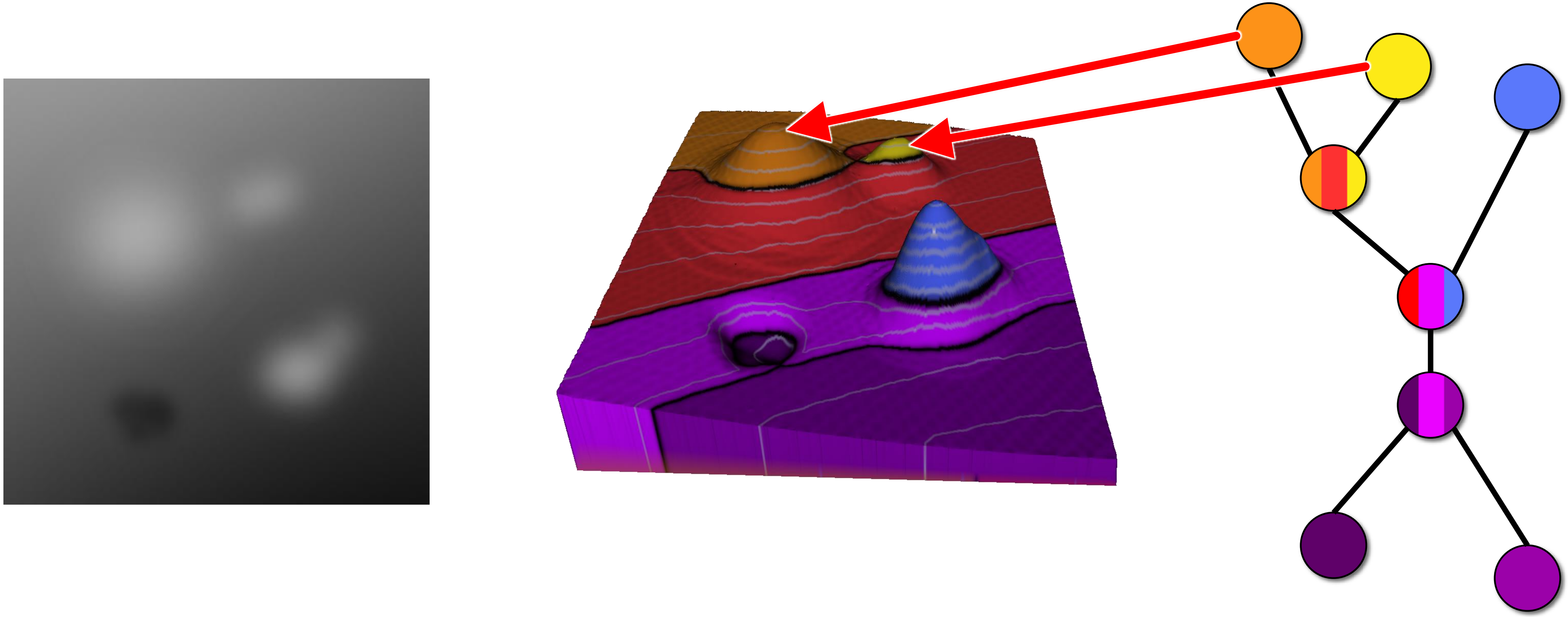






Contour trees



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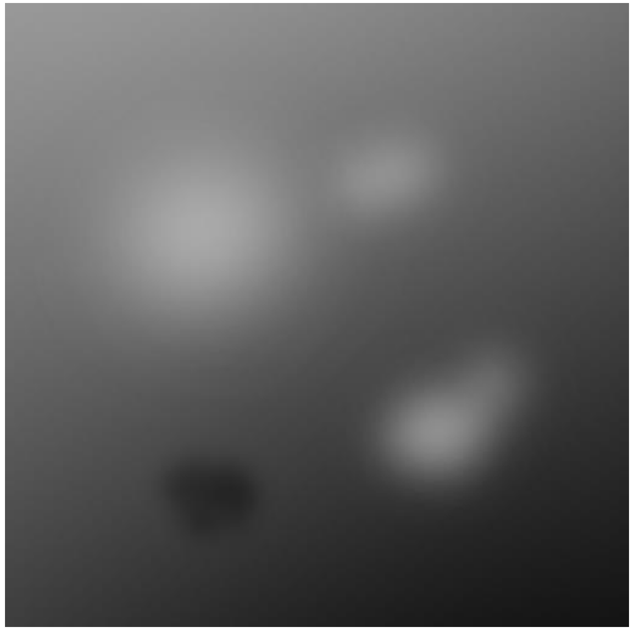
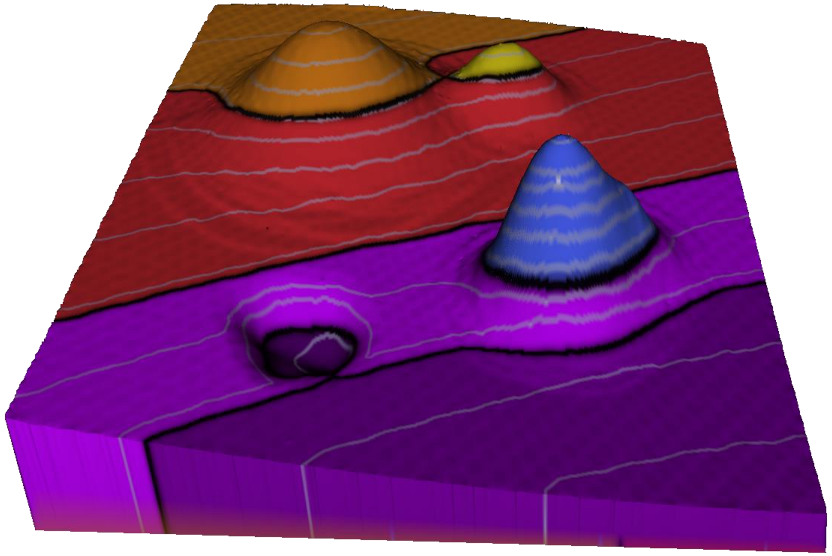
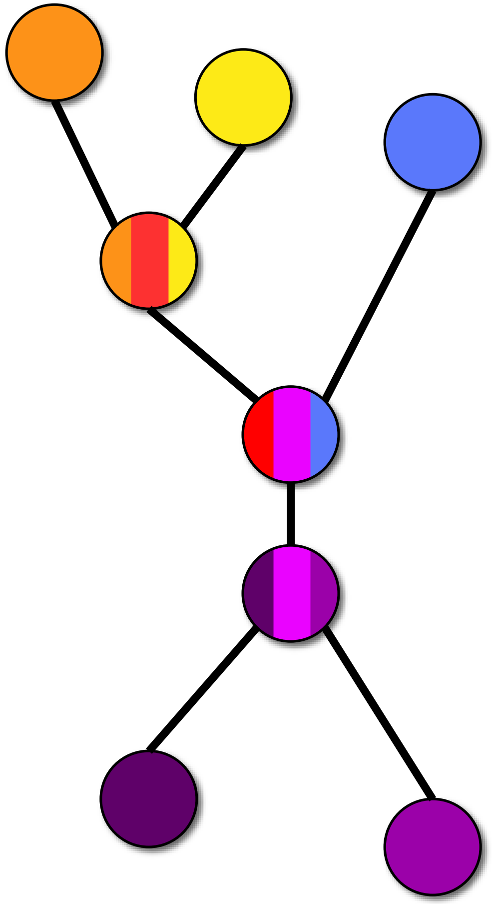
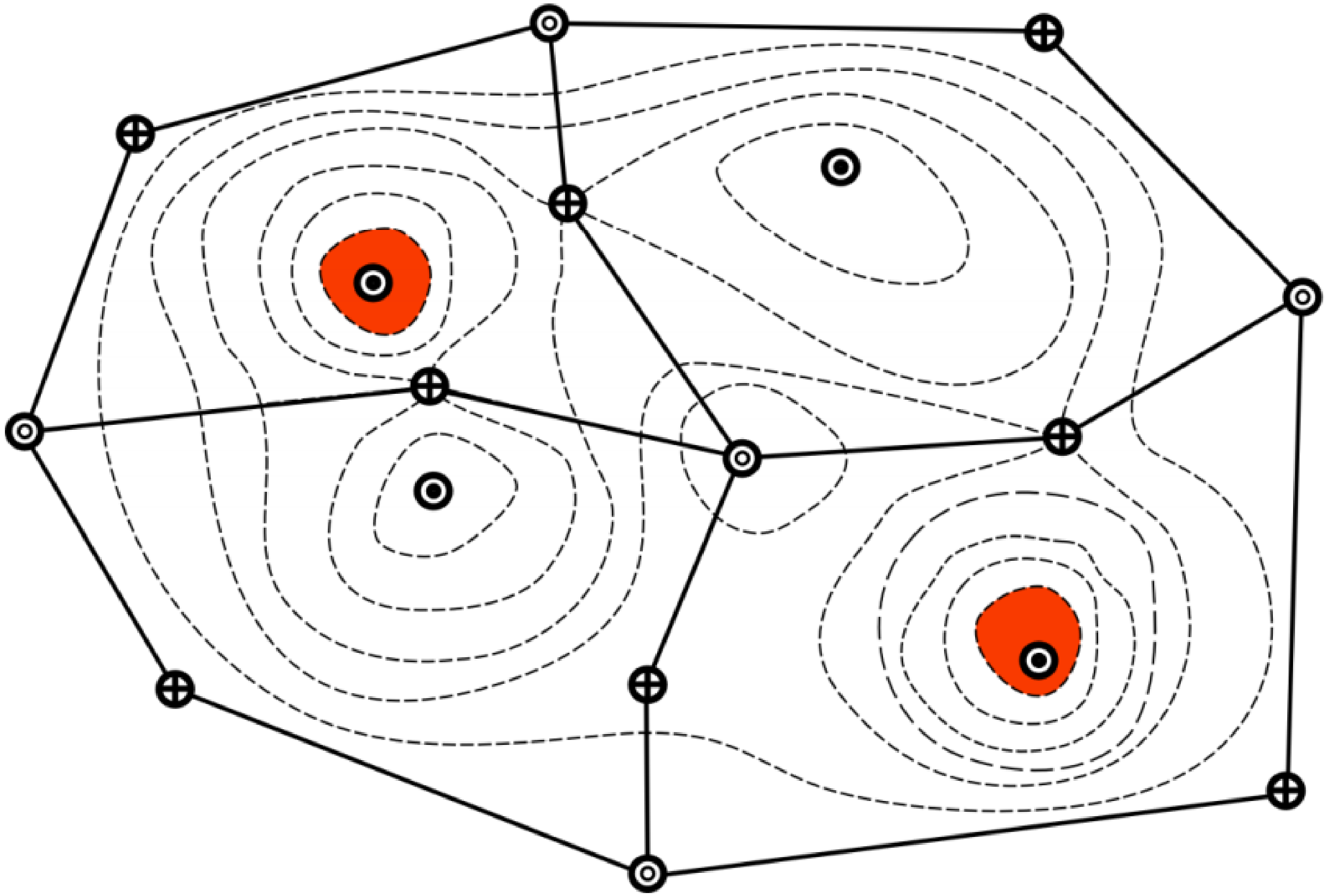
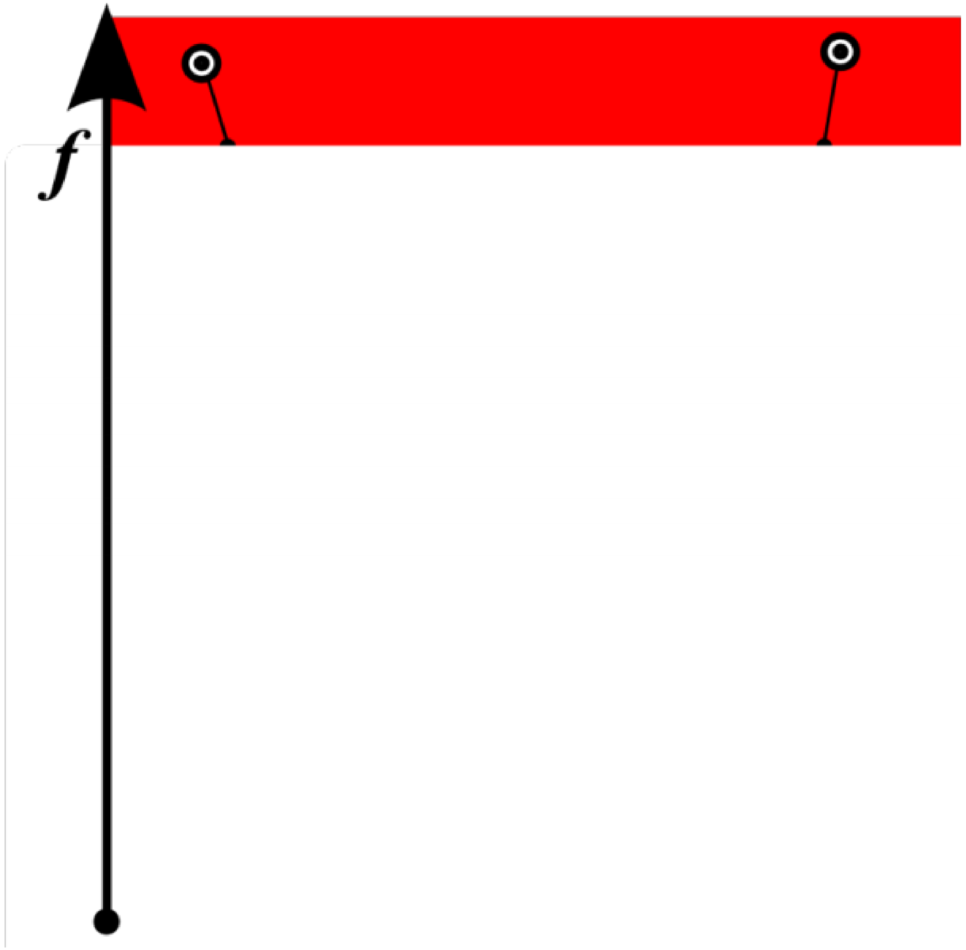






A Closer Look at the Contour tree





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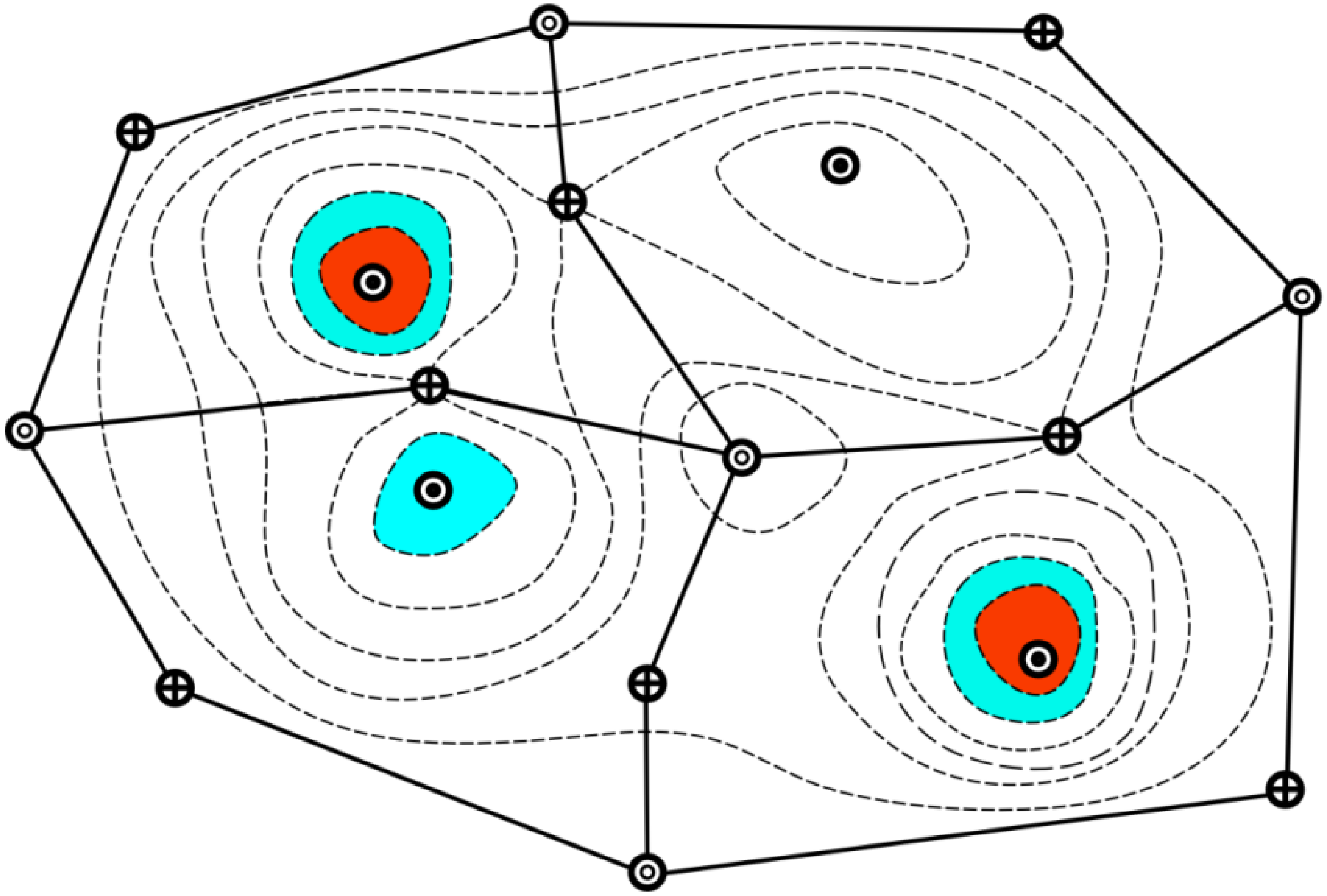

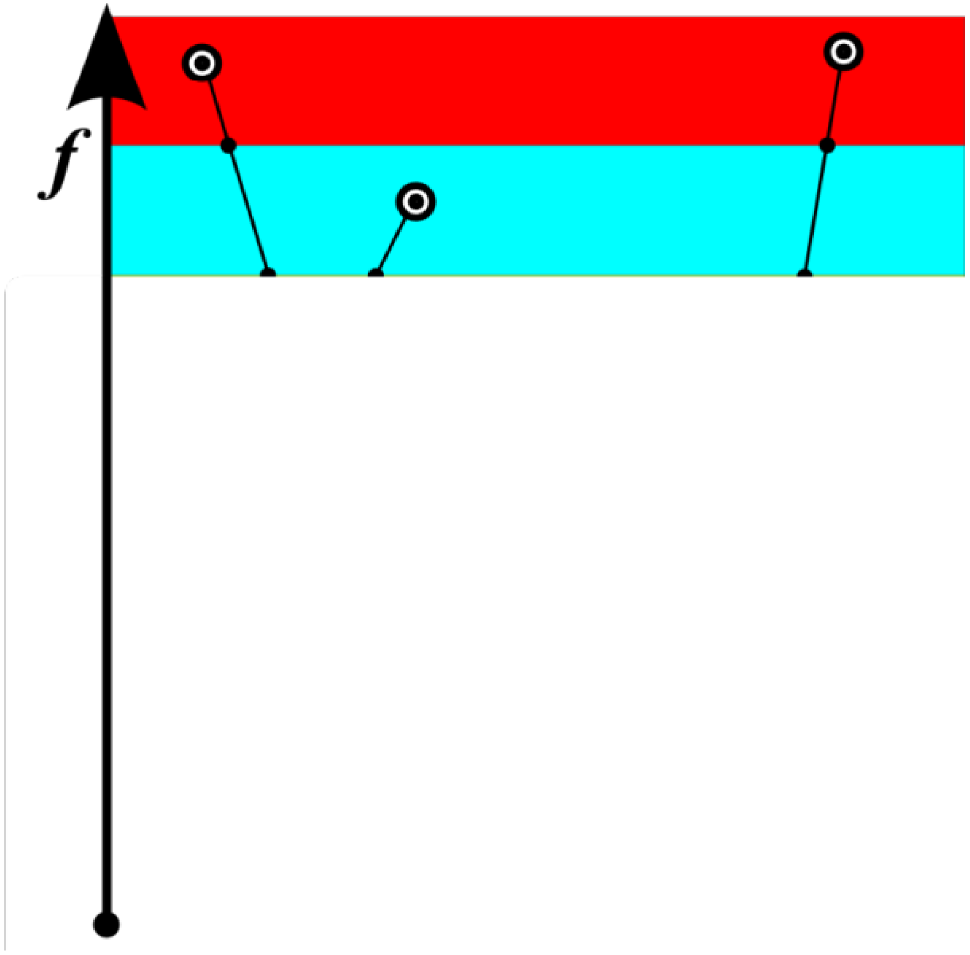

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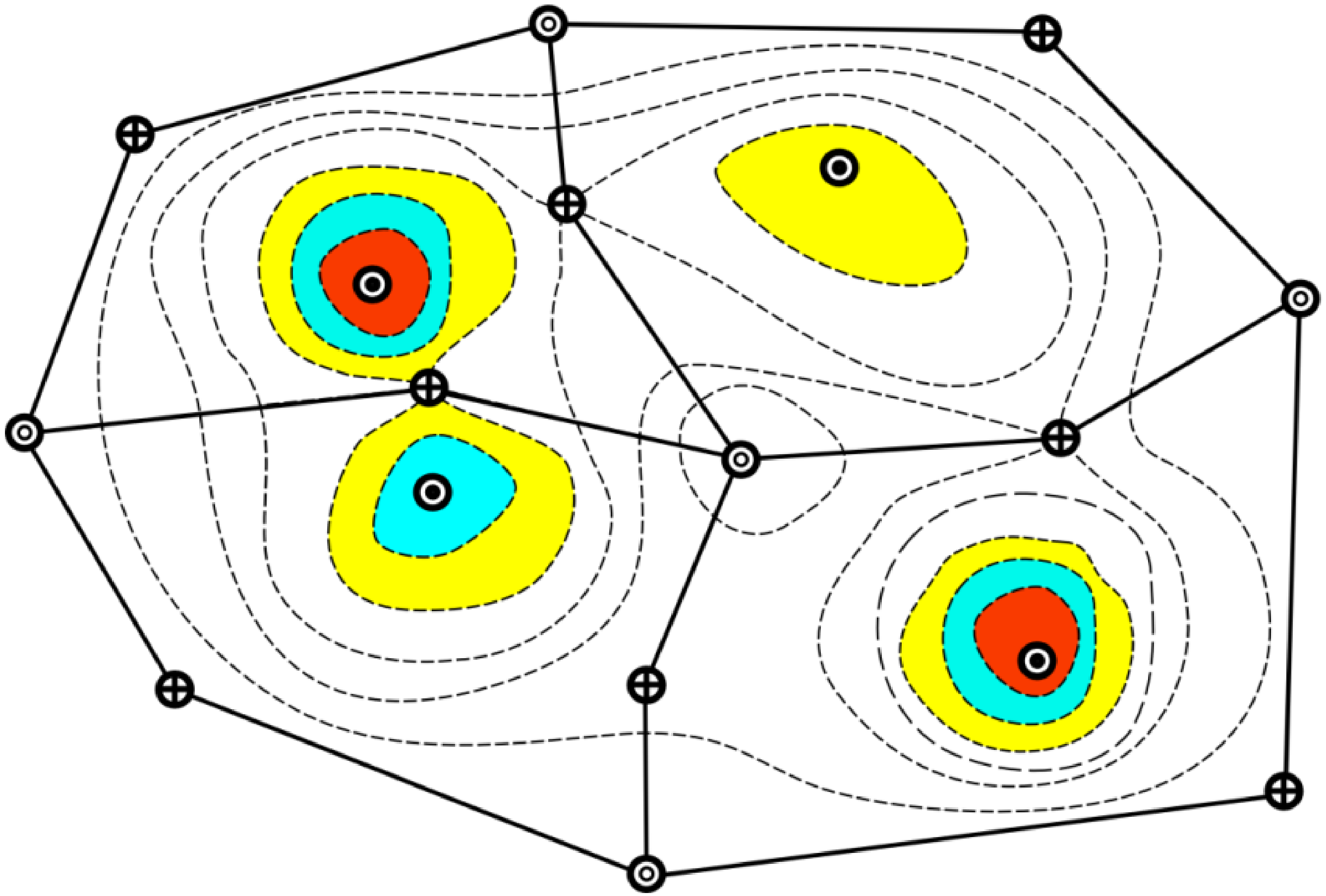
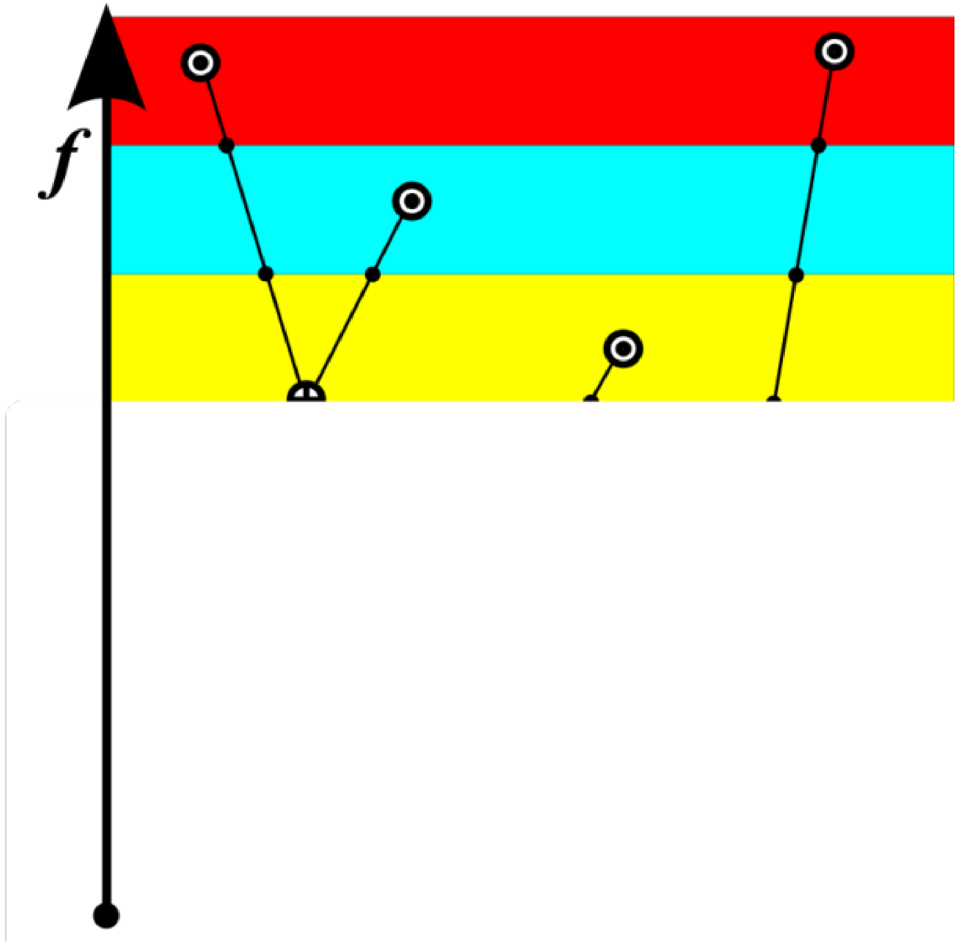
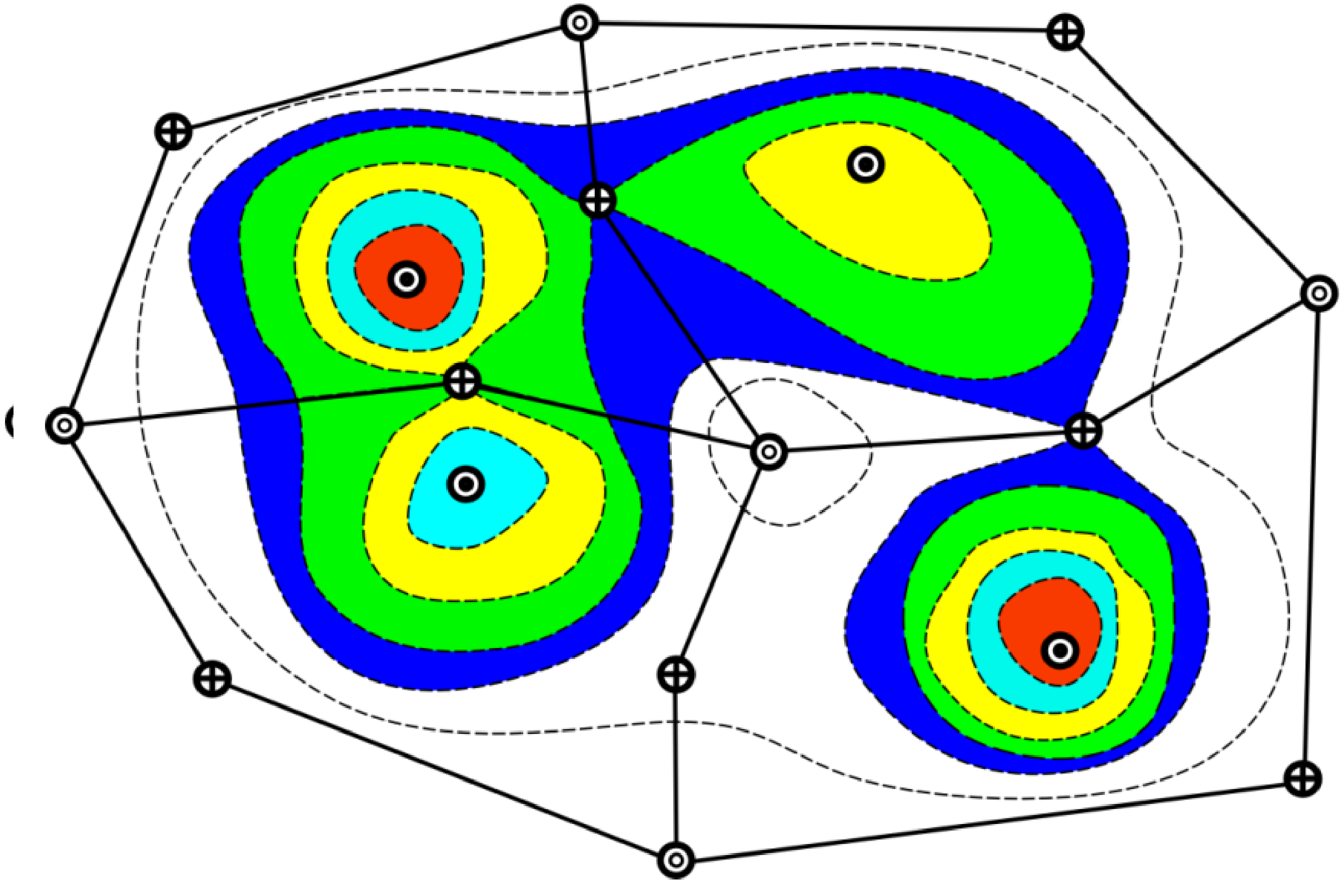
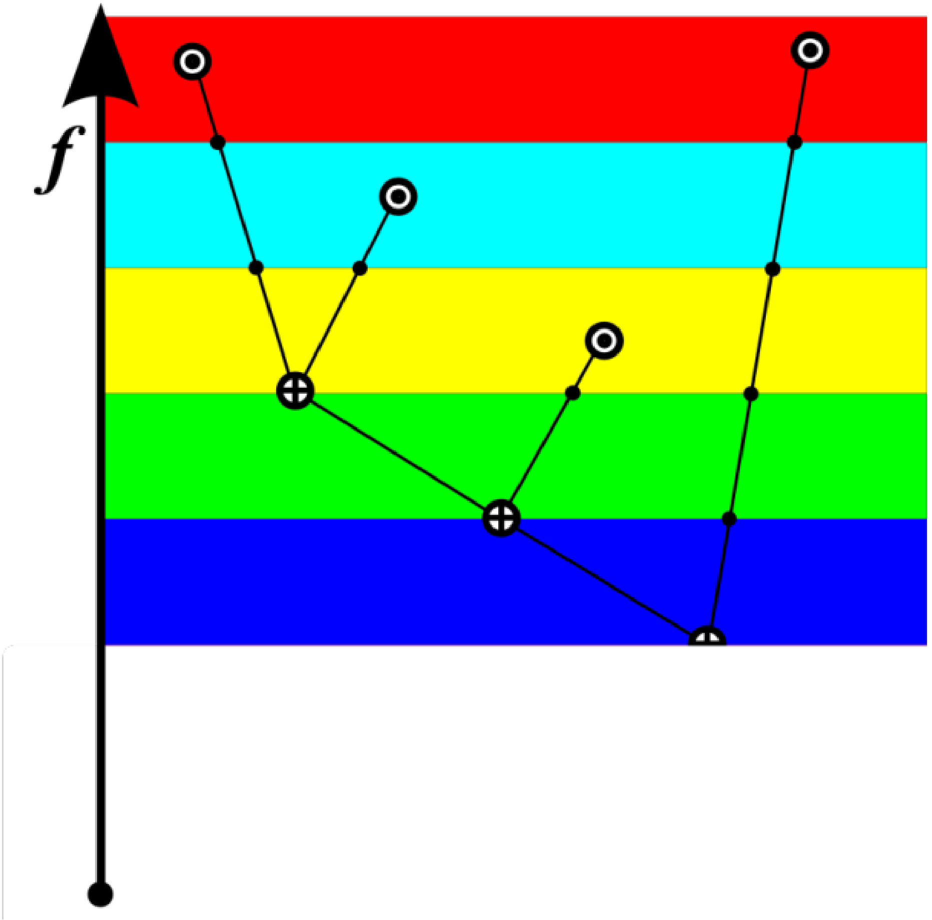




A Closer Look at the Contour tree





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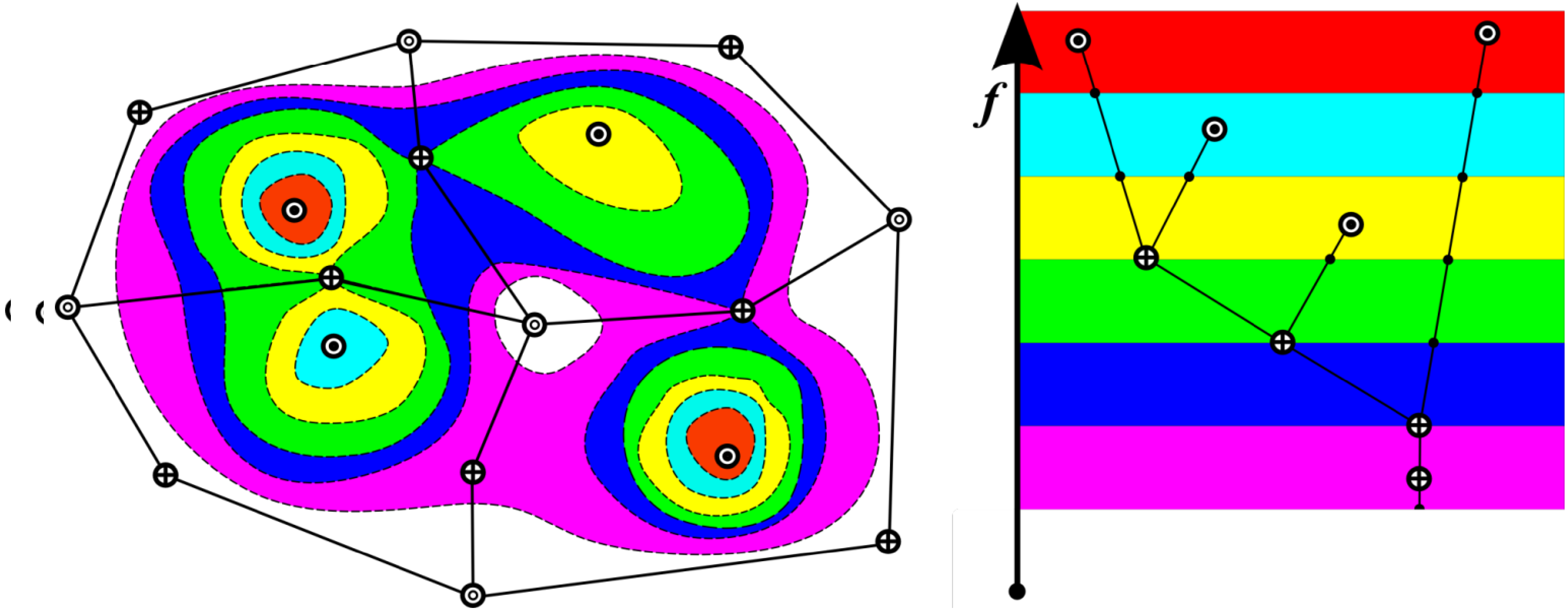

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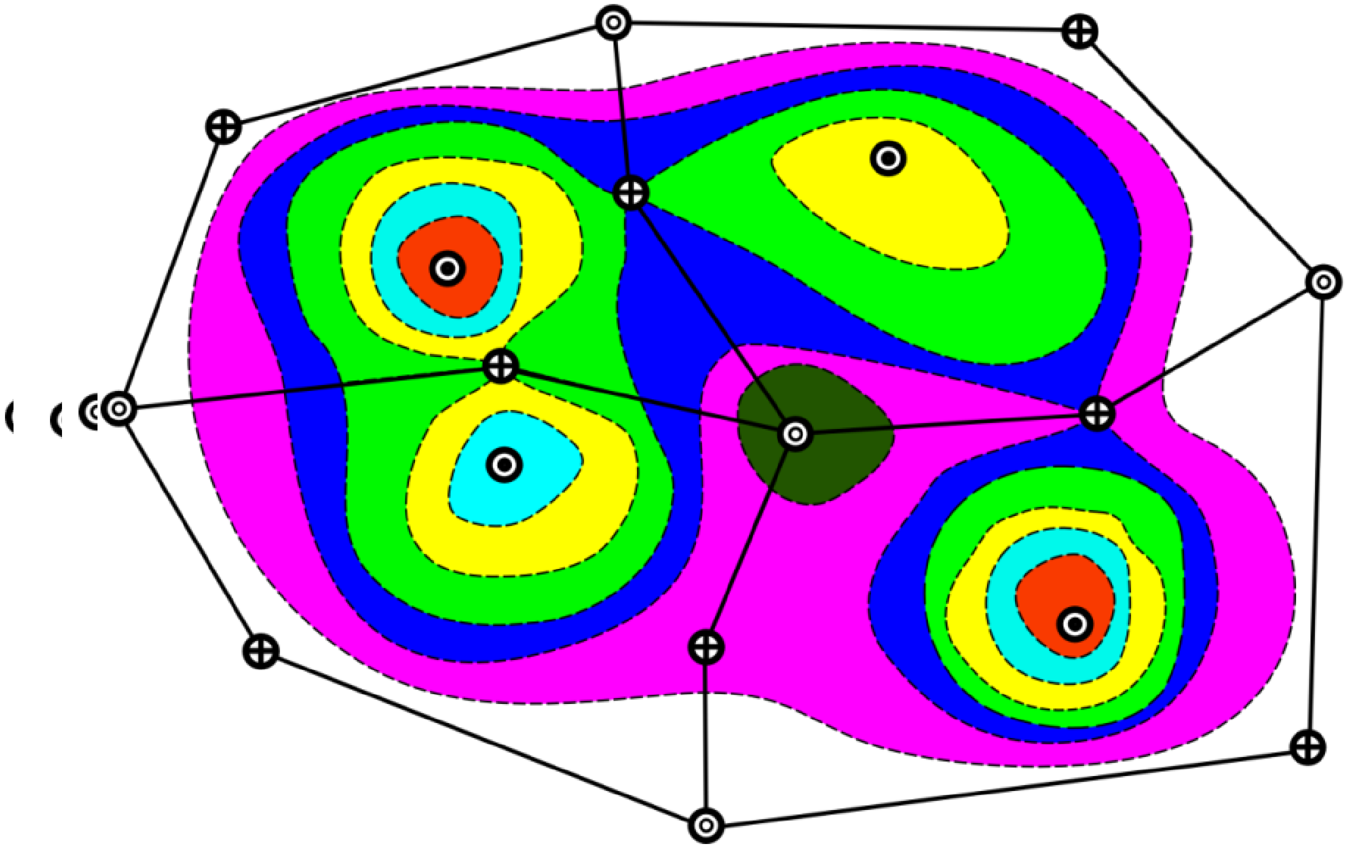
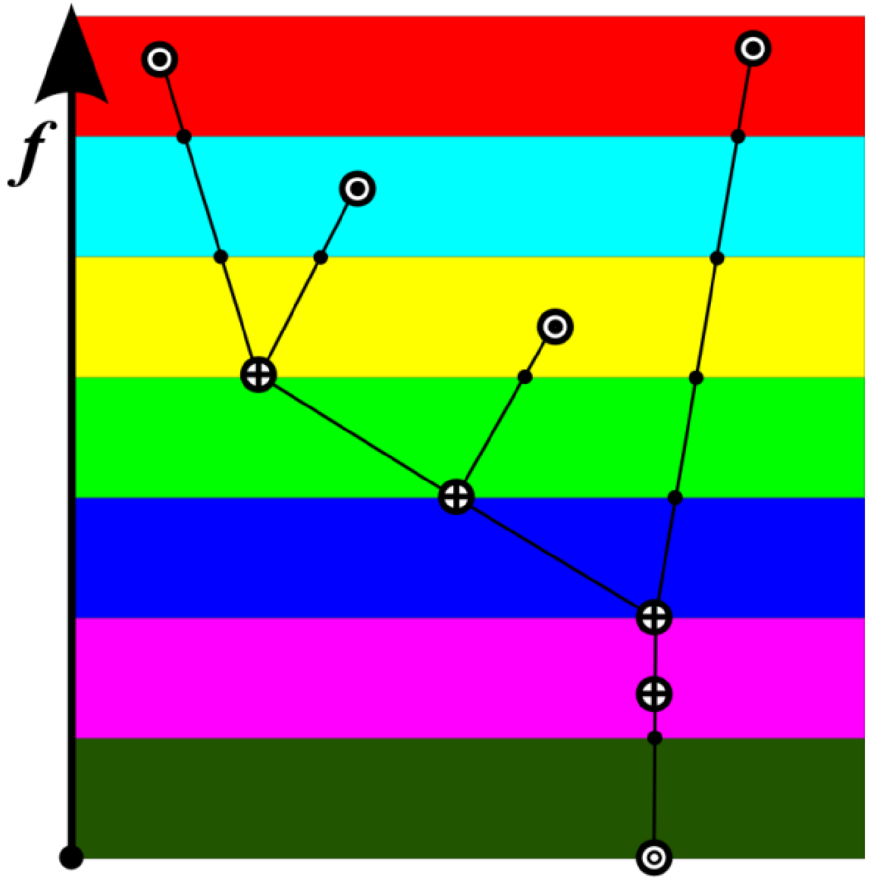
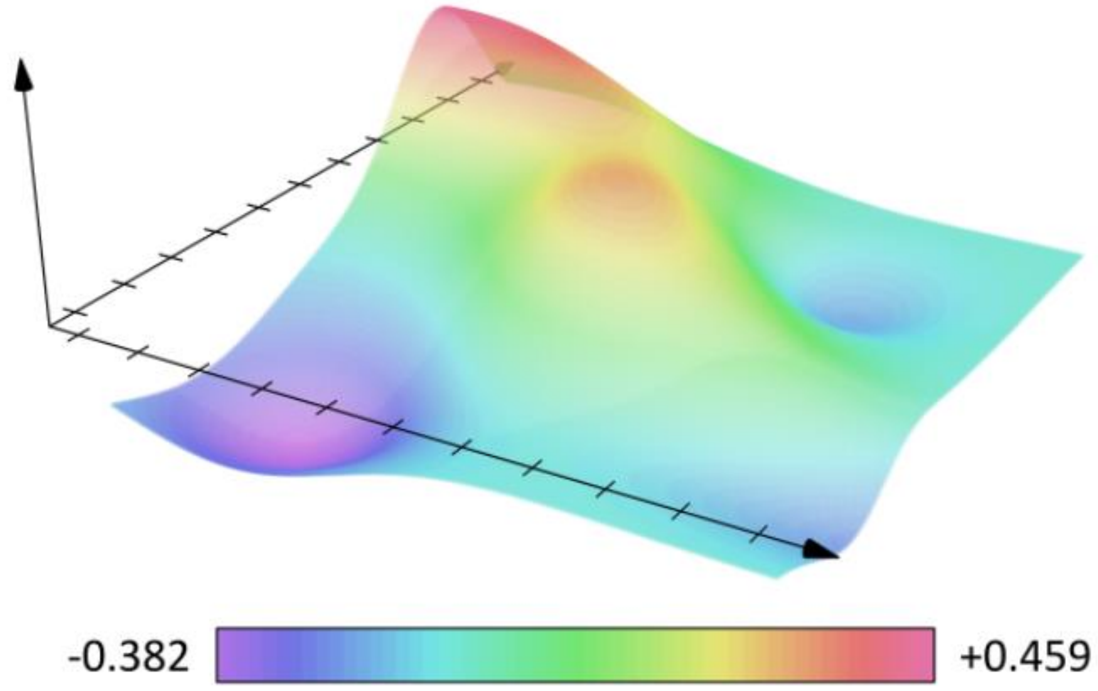
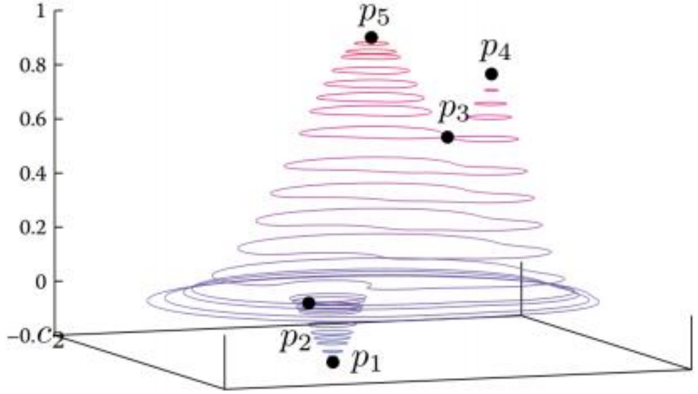
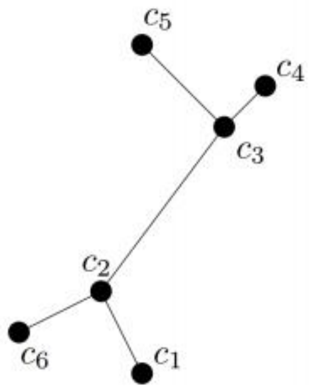




A Closer Look at the Contour tree







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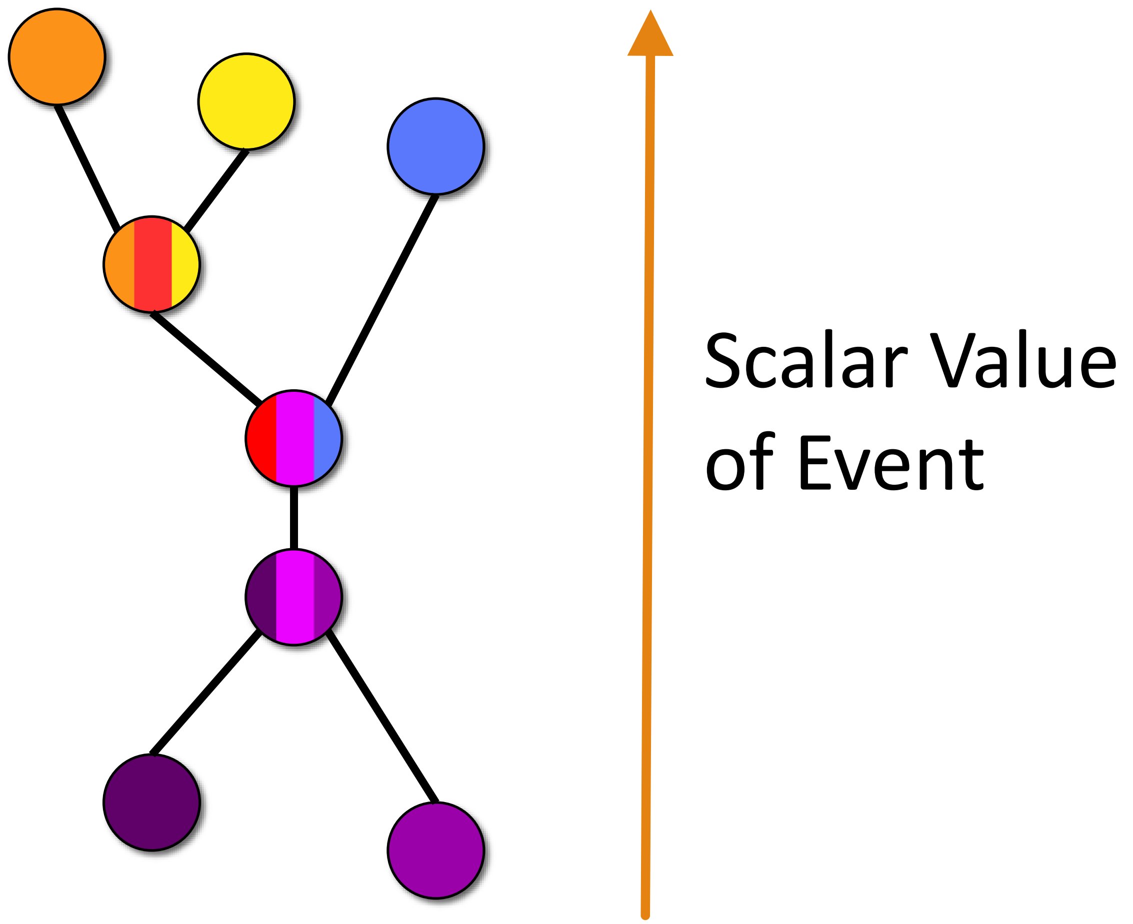
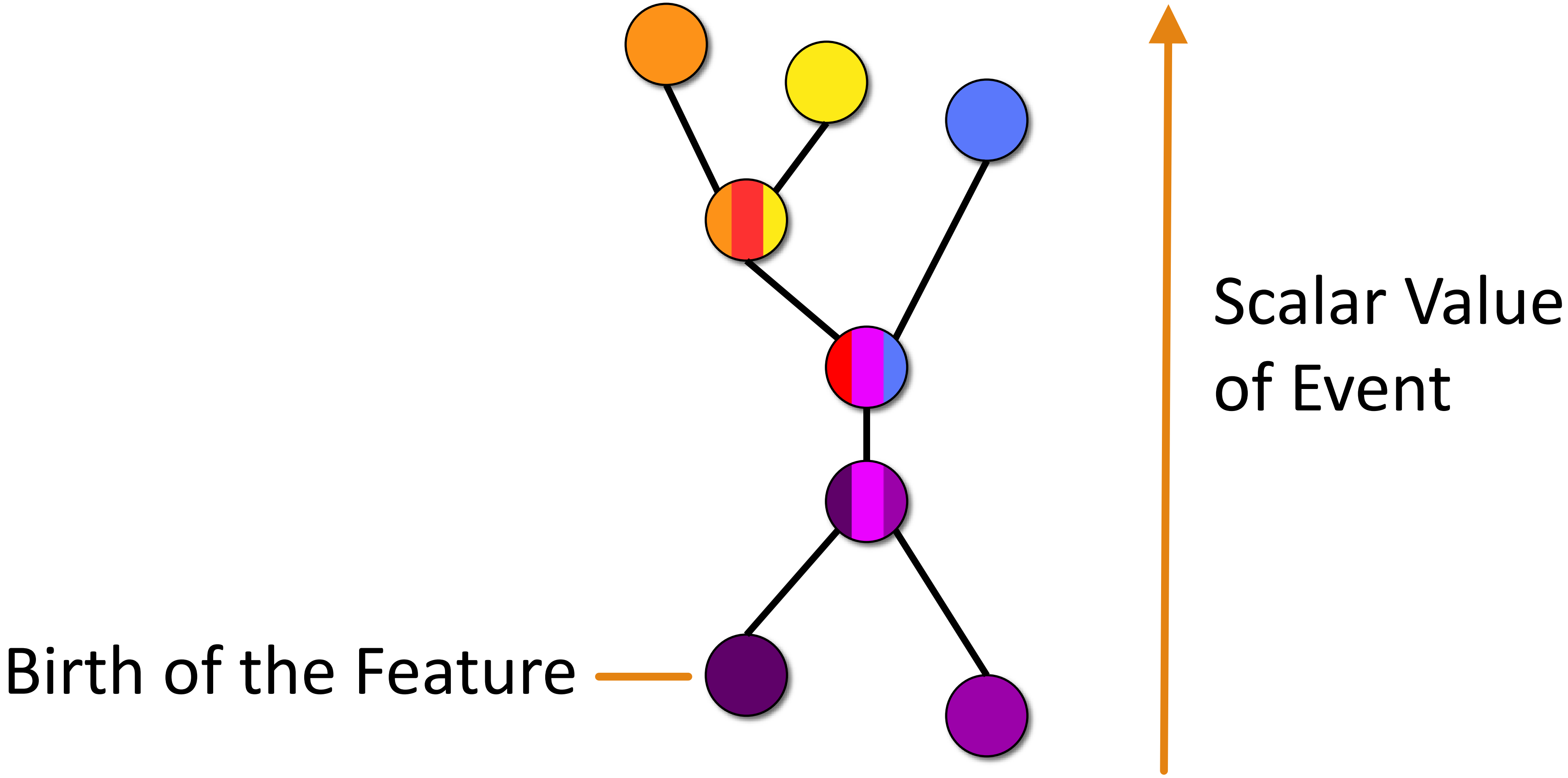


A CLOSER LOOK AT THE CONTOUR TREE



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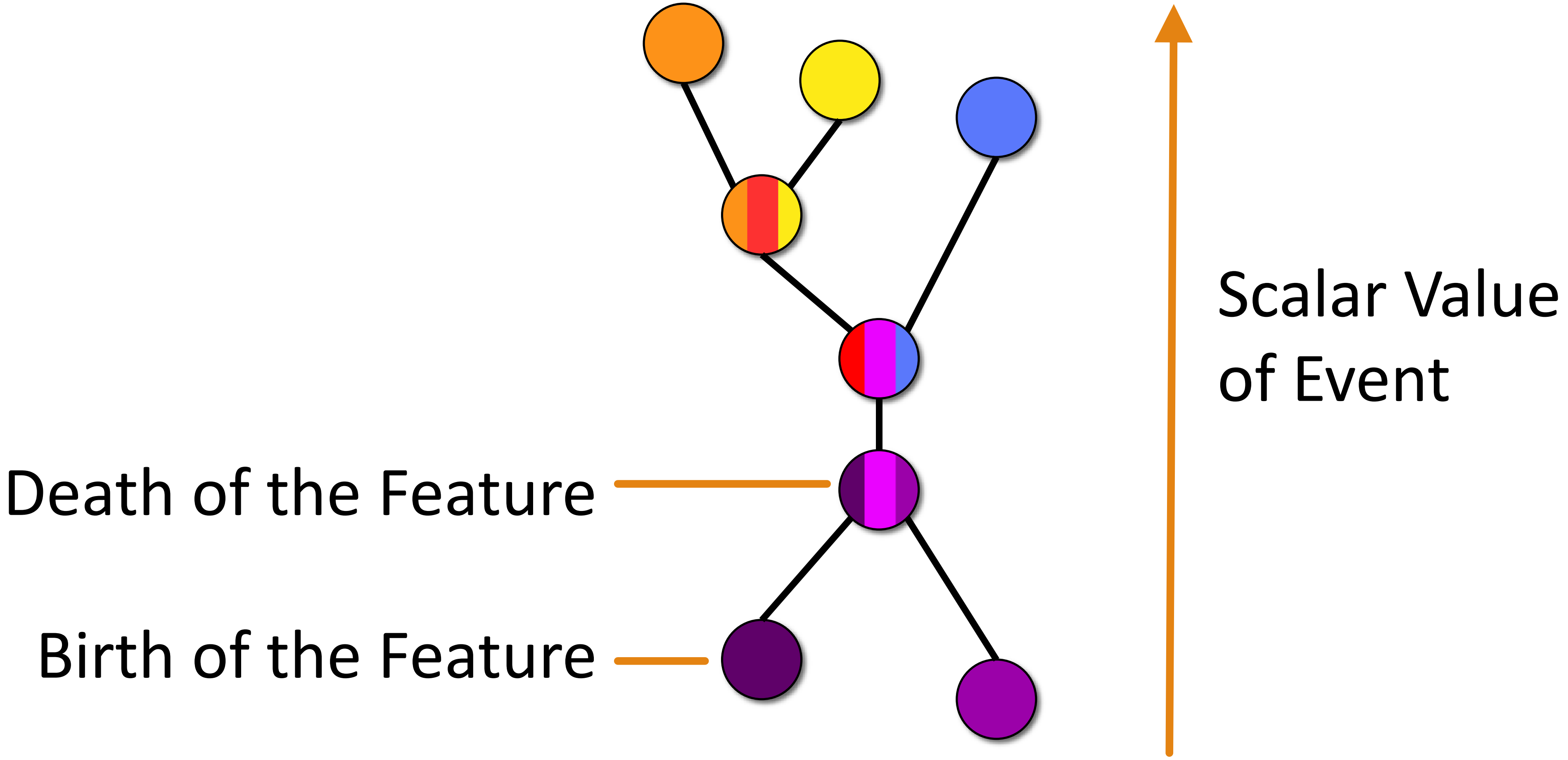

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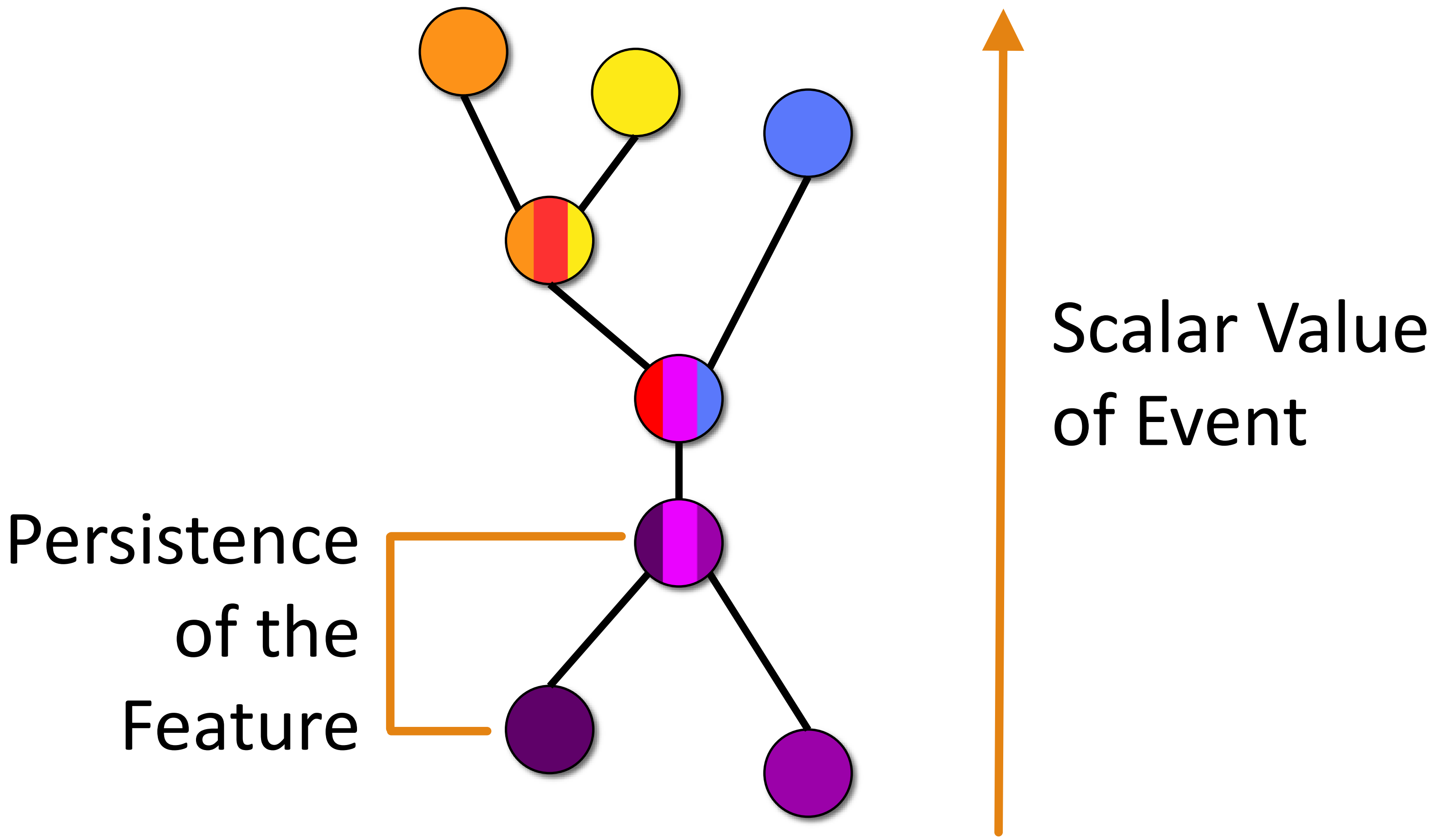
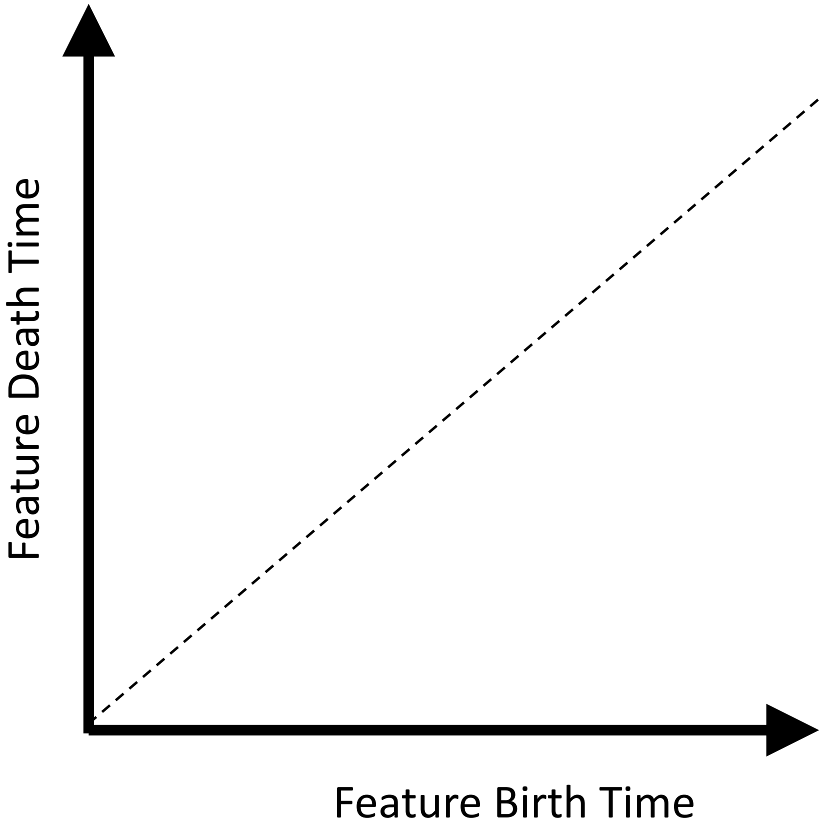
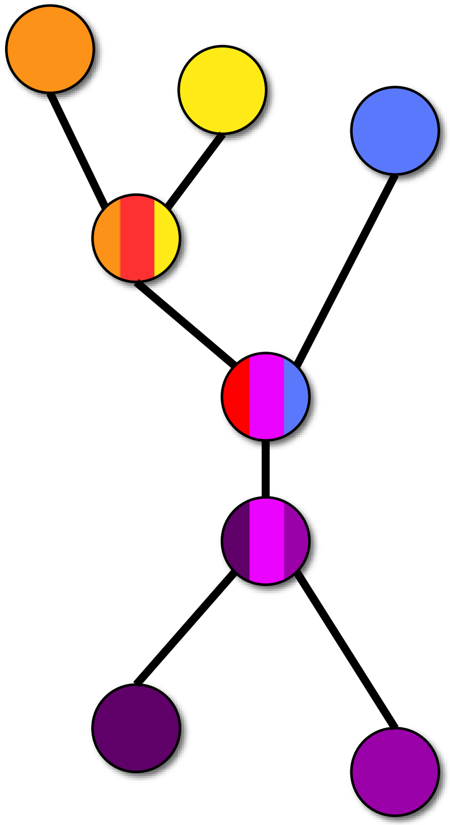


Controlling Simplification
The Persistence Diagram





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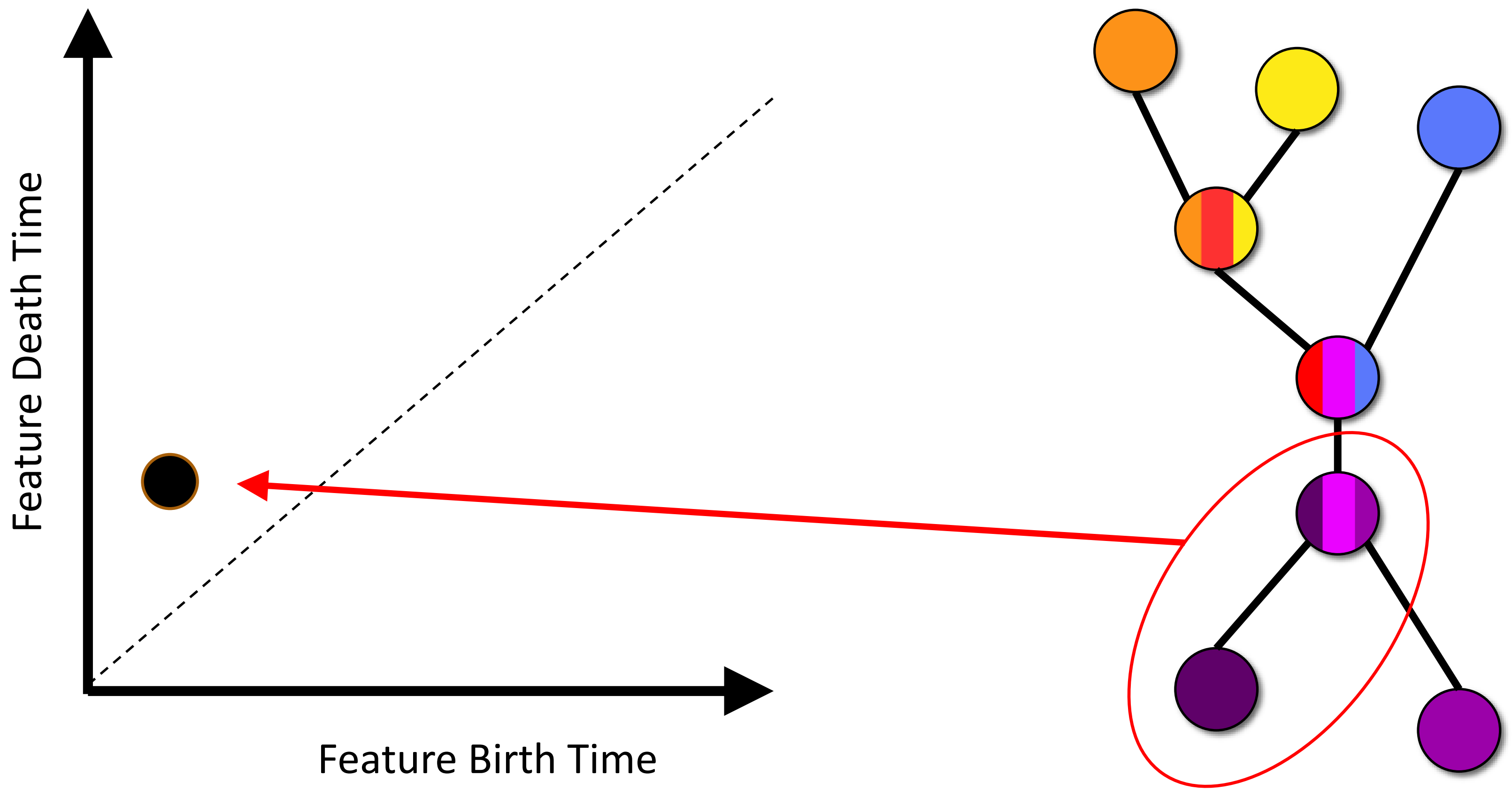

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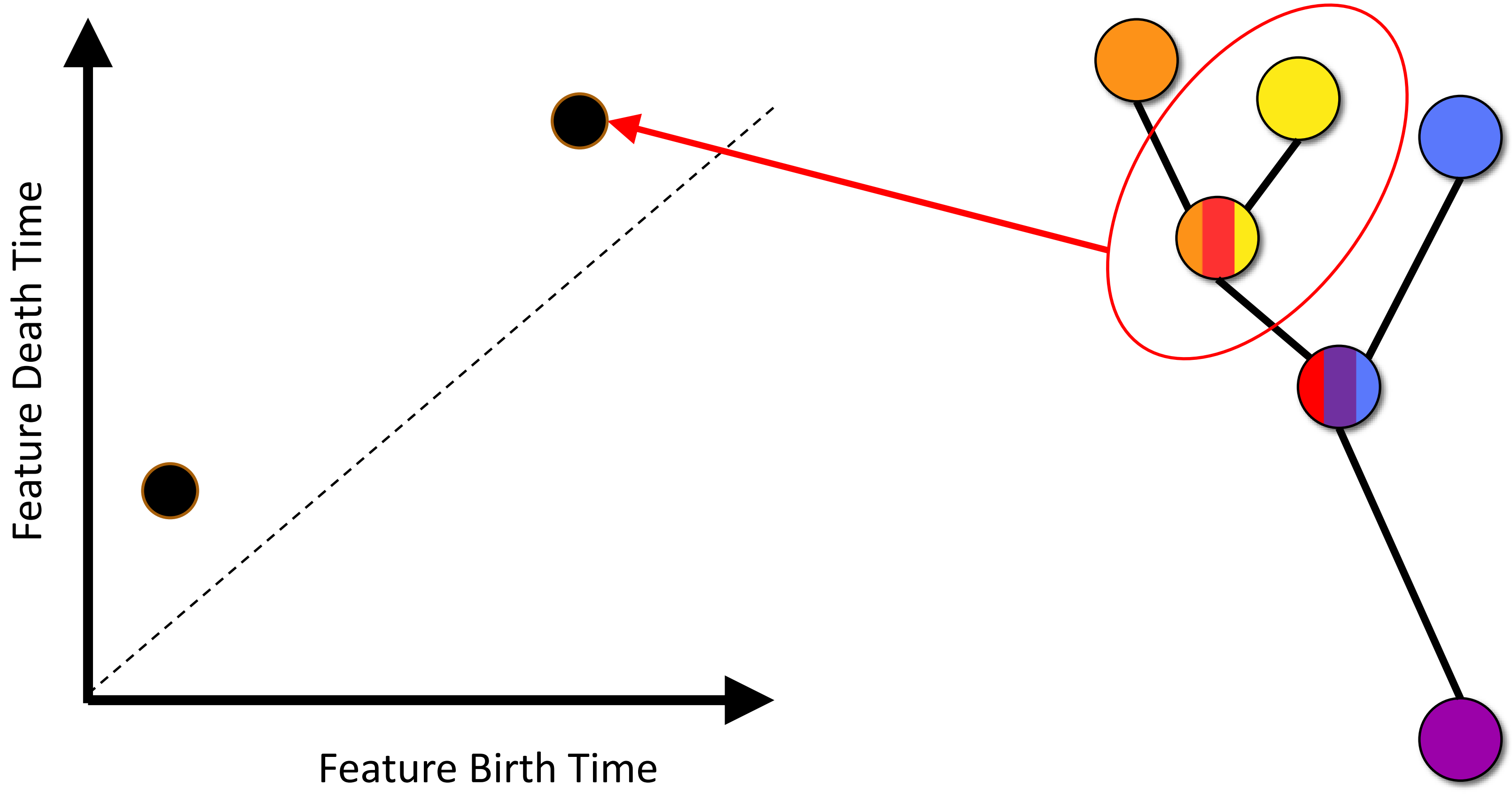

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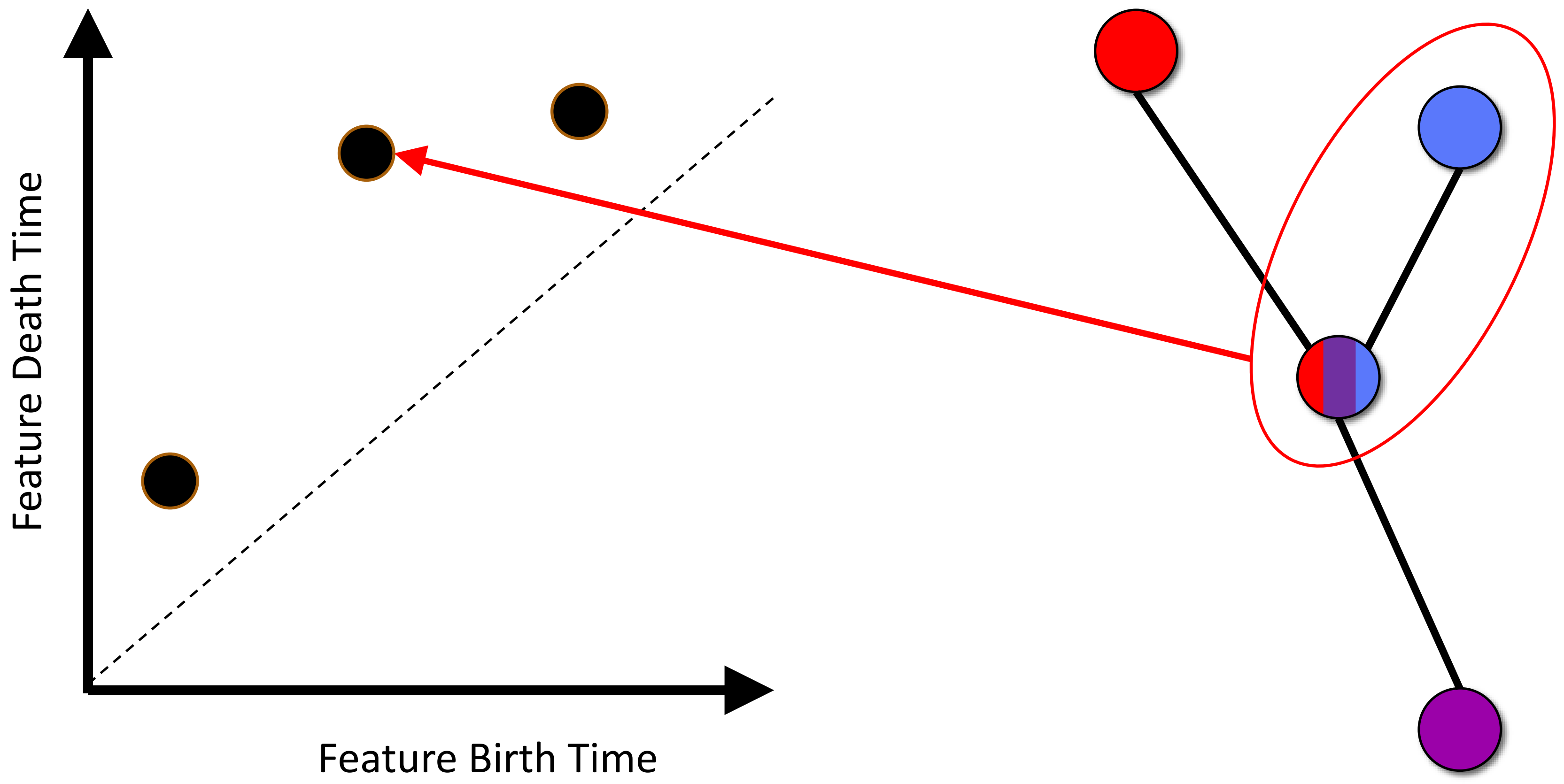

Controlling Simplification

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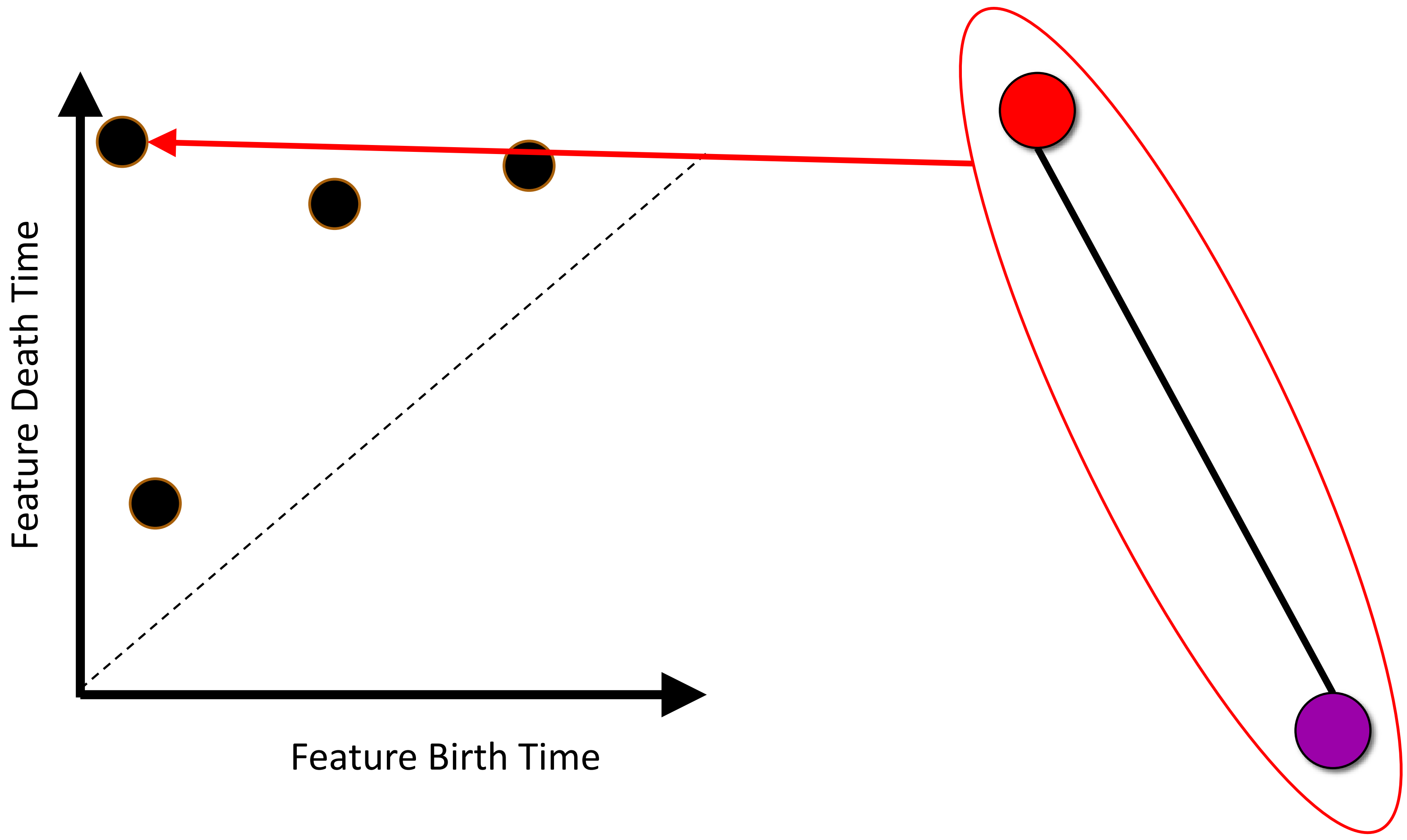

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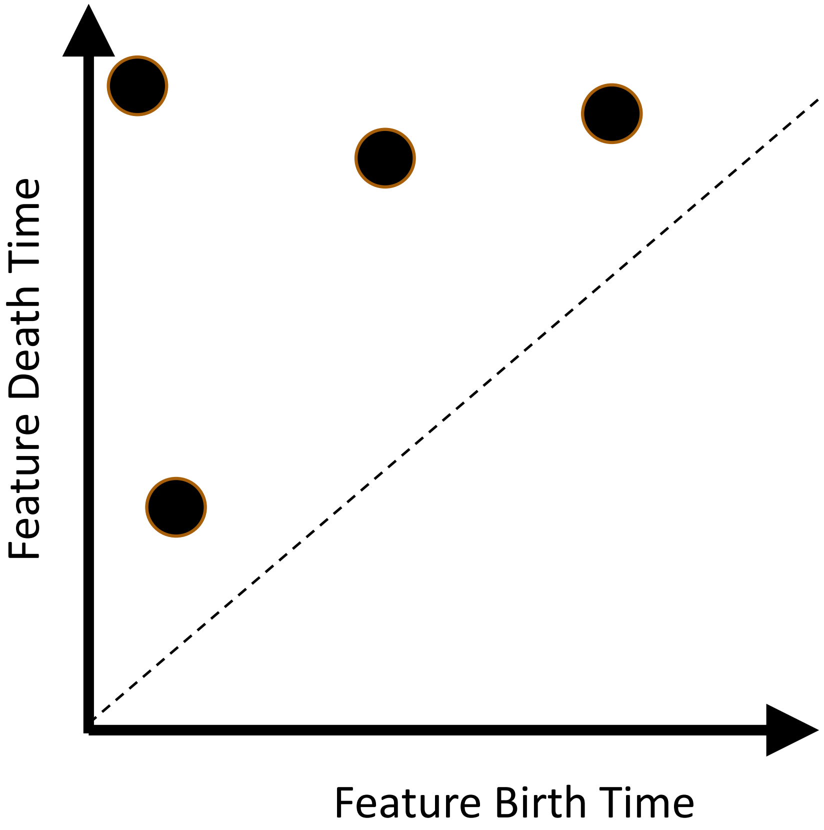

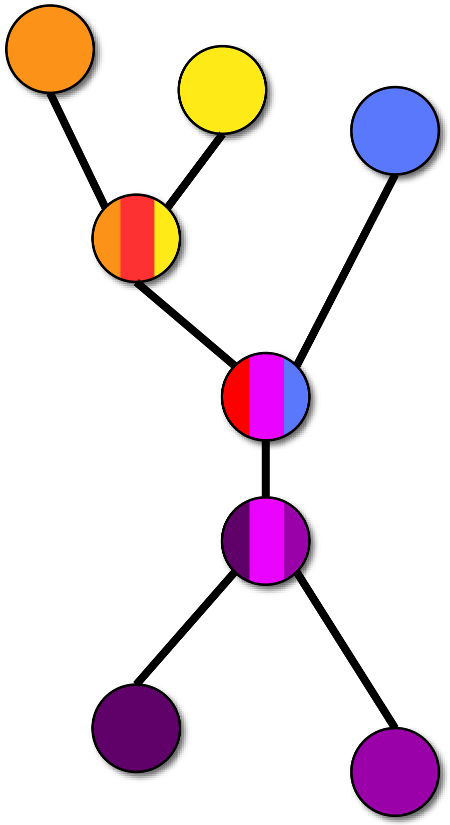

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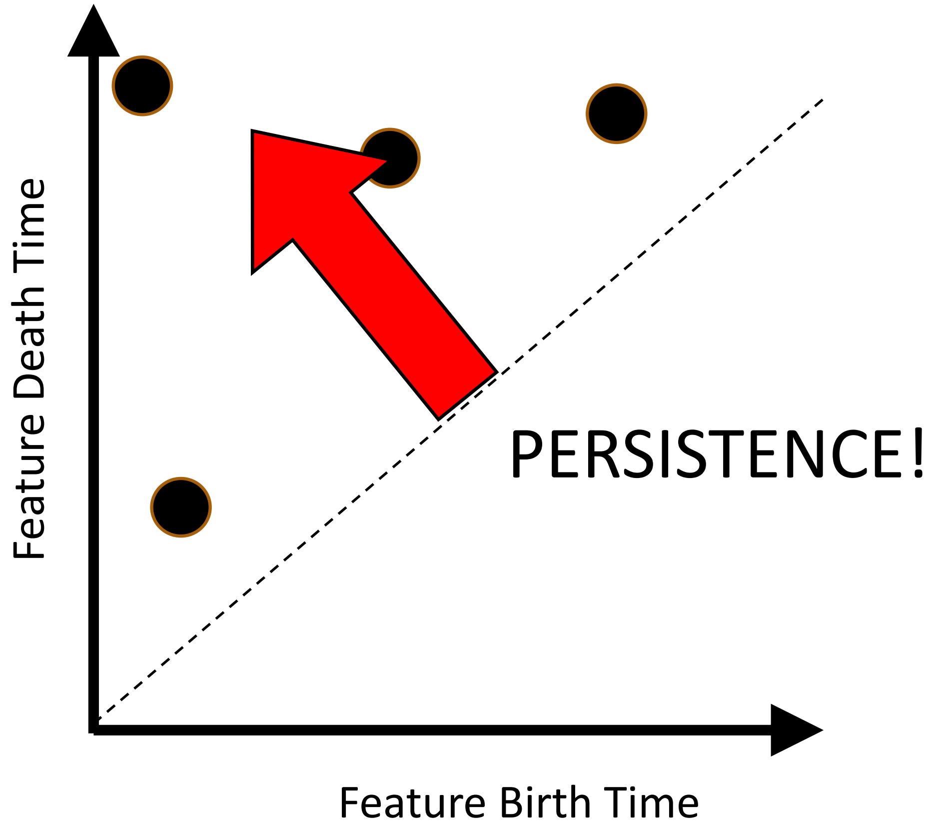
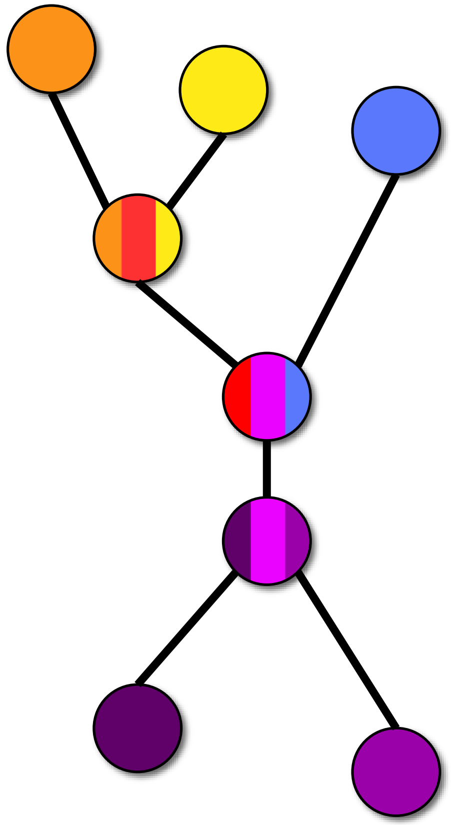
The Persistence Diagram





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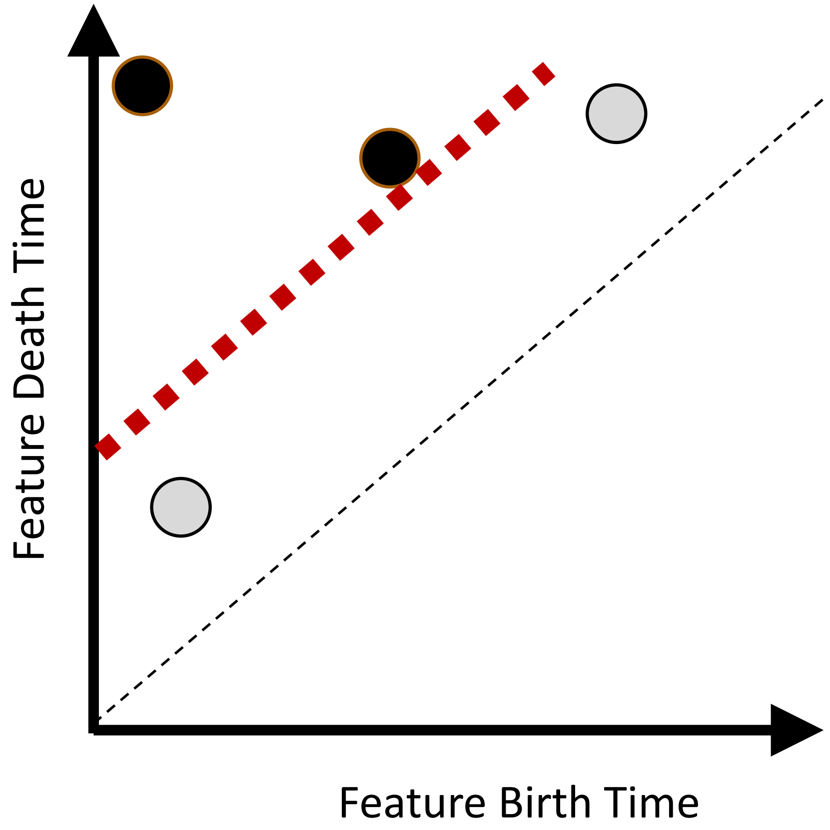
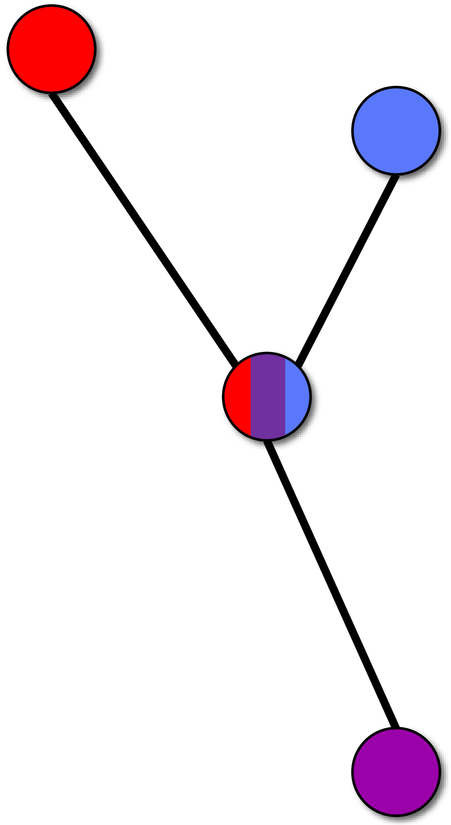
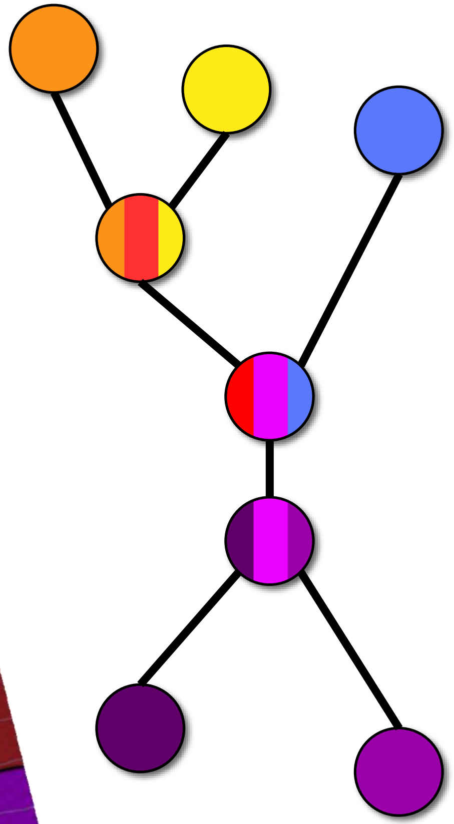
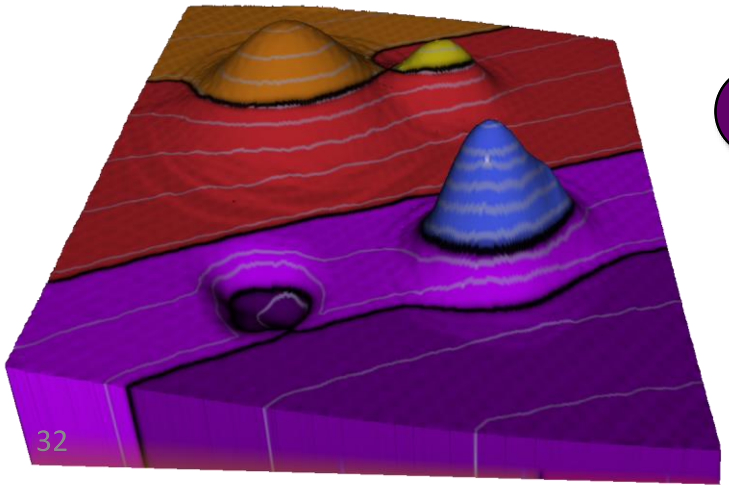




Feature Removal





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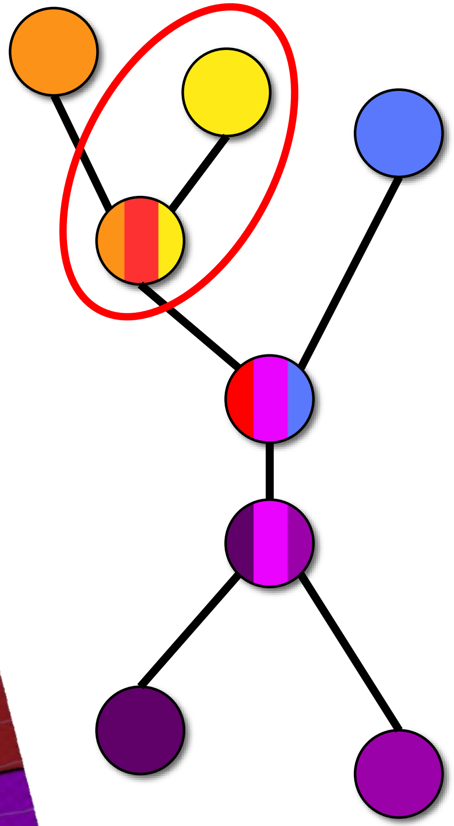

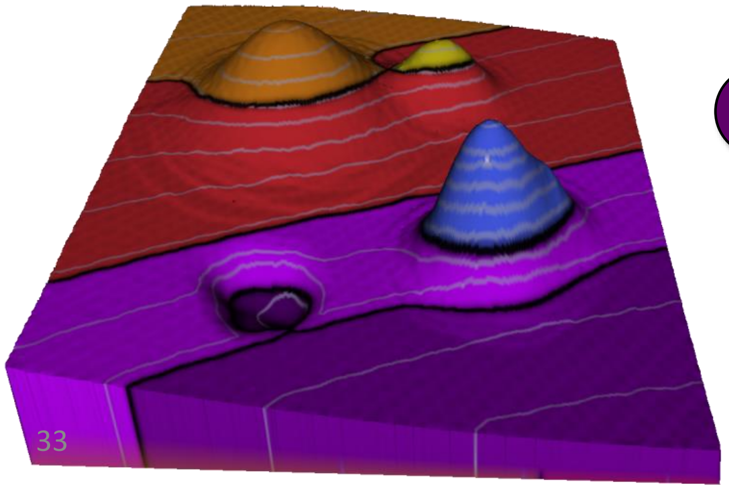

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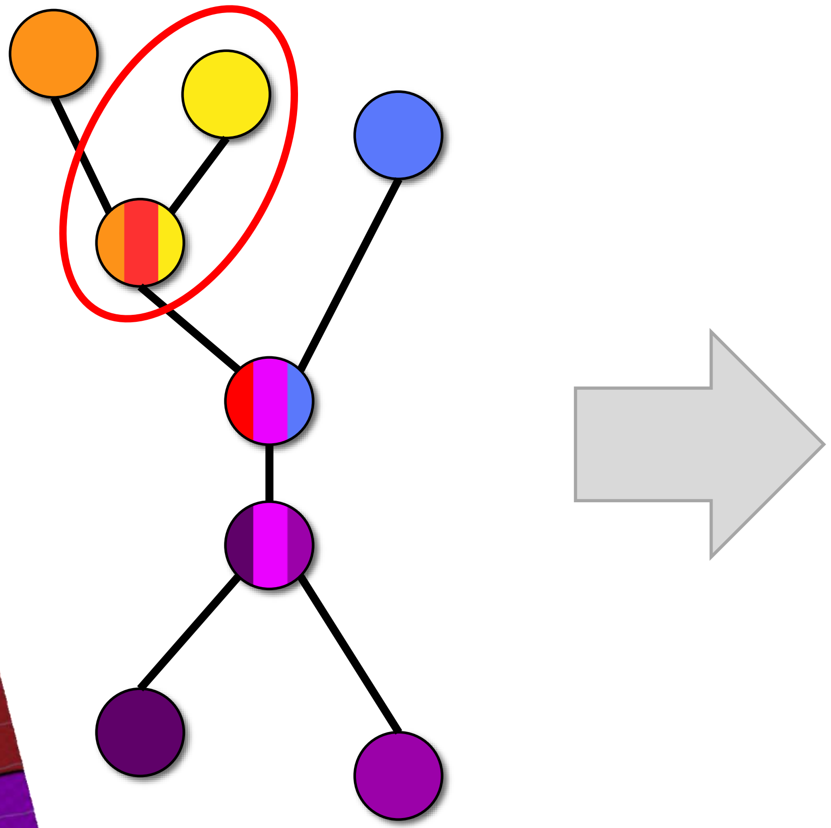

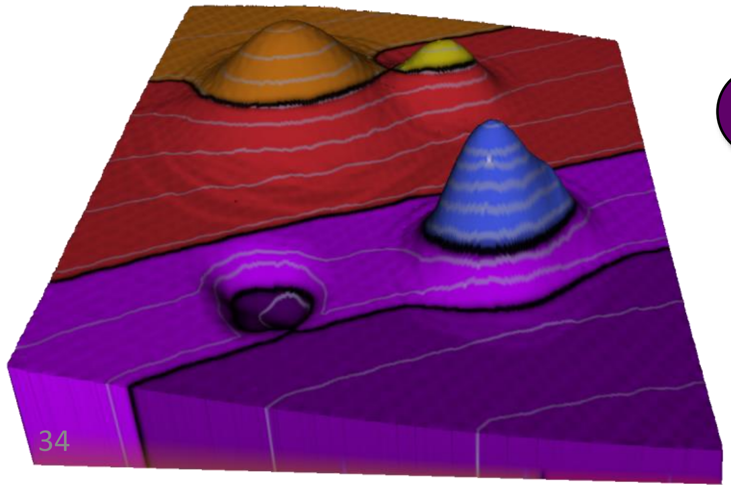

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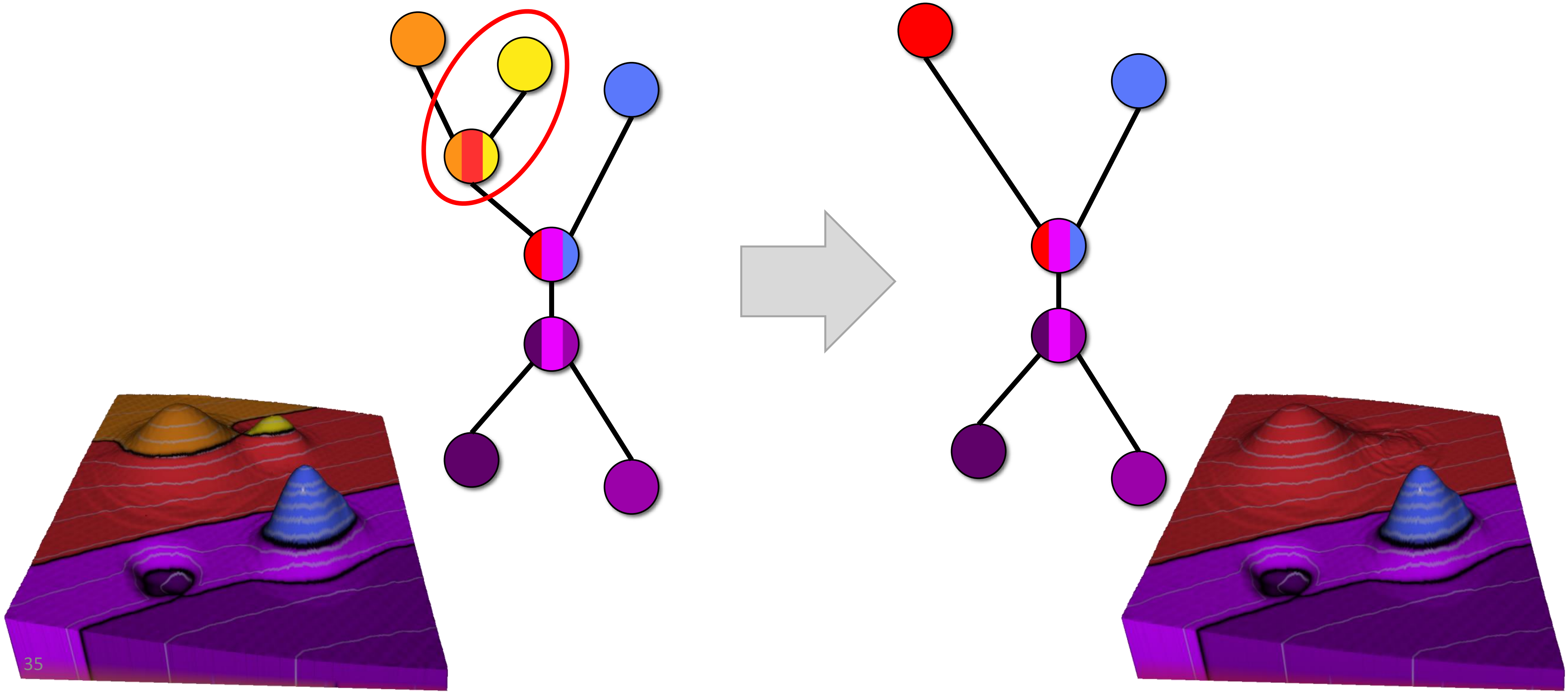
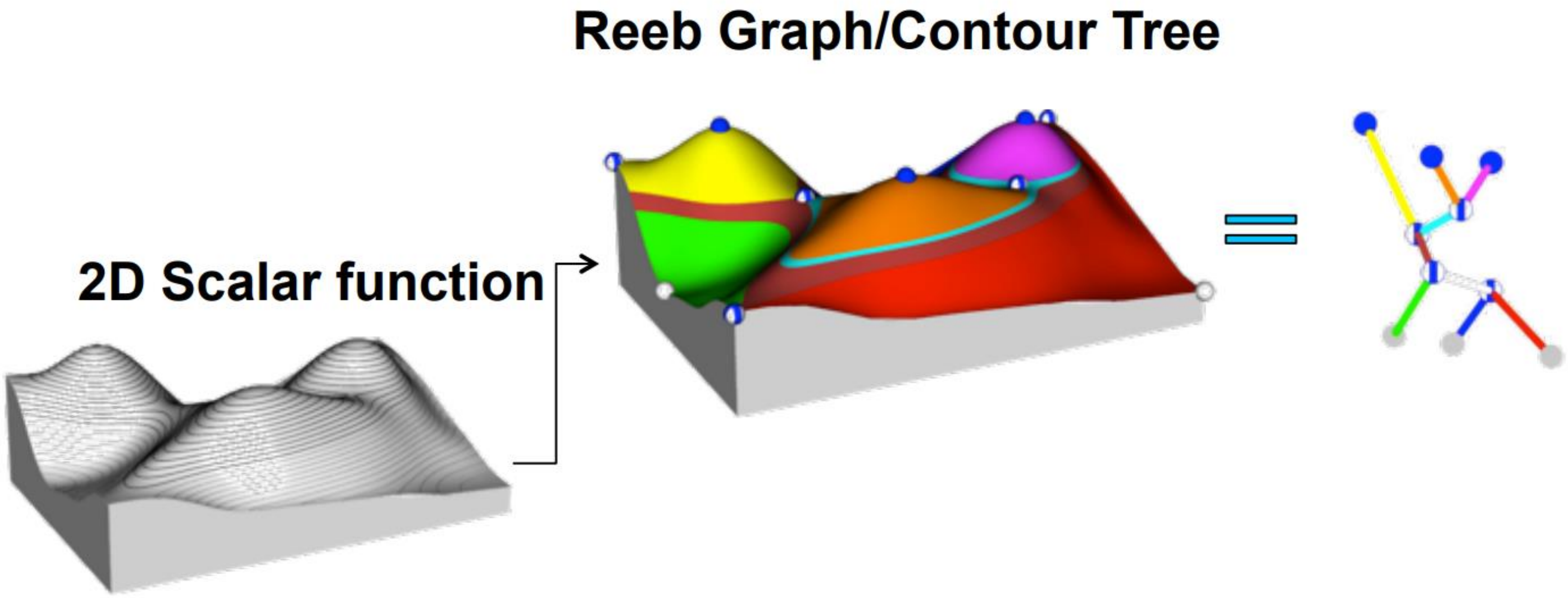
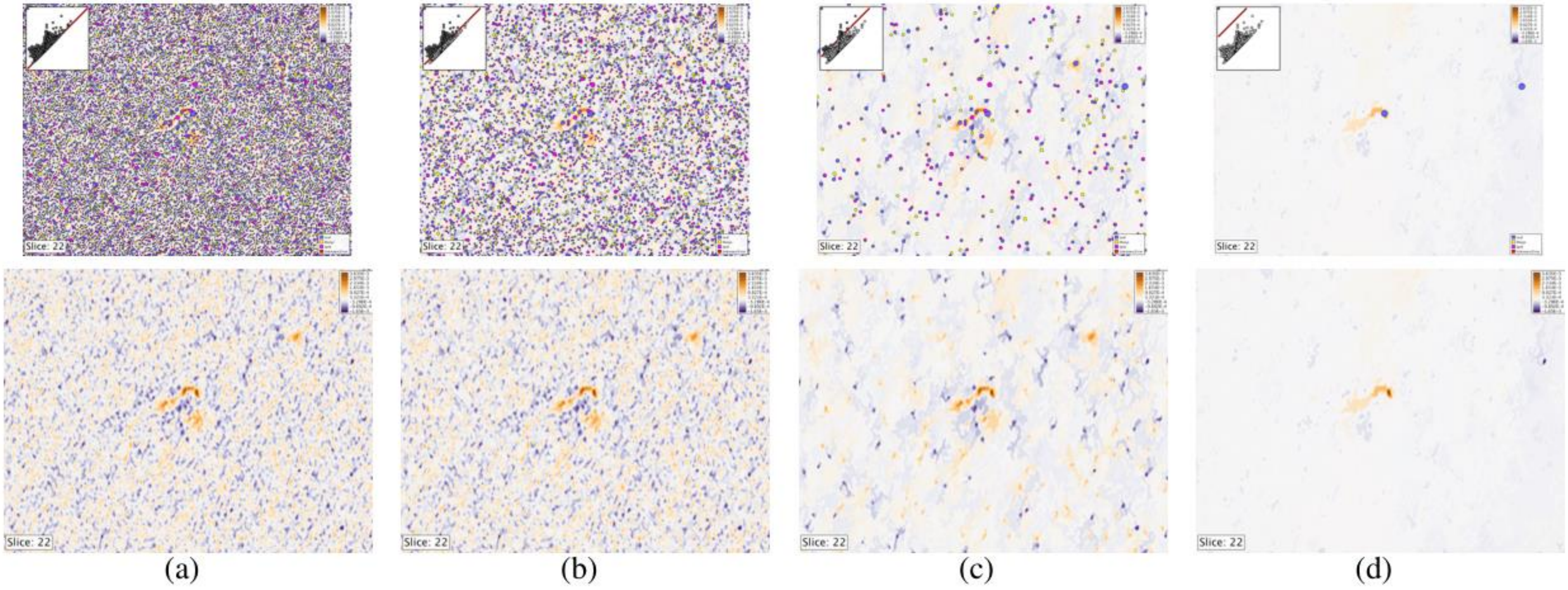



Reeb Graph/Contour Tree
2D Scalar function

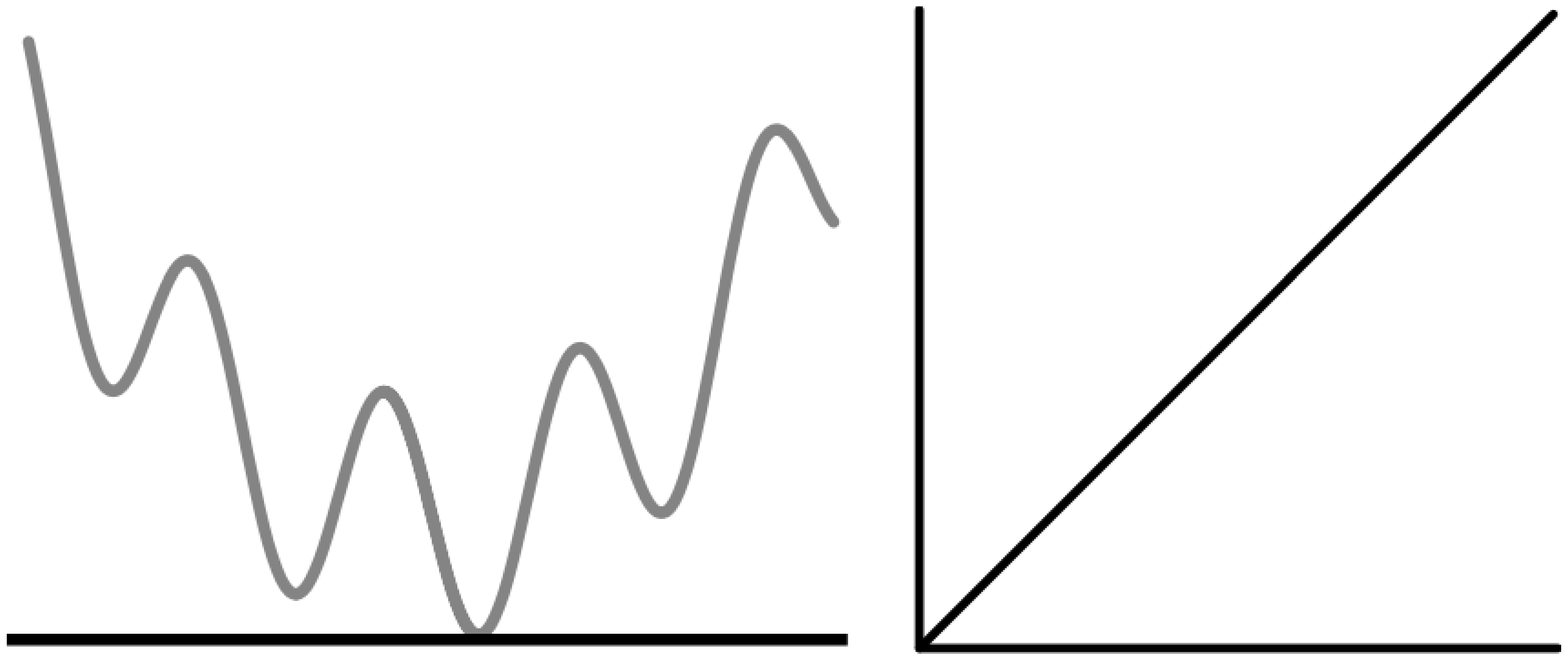
Application



Example Contour Tree simplification of radio astronomy data. Top: The critical points of the Contour Tree are overlaid on the scalar field. Bottom: The simplified scalar field is colored with a divergent colormap, blue for negative and red for positive. The simplification level goes from none on the left (a) to very aggressive on the right (d).

Persistence Diagram of a scalar function

- Track the evolution of the topology of sub-level sets as the threshold increases.
- Pair thresholds that create components with those that destroy them.



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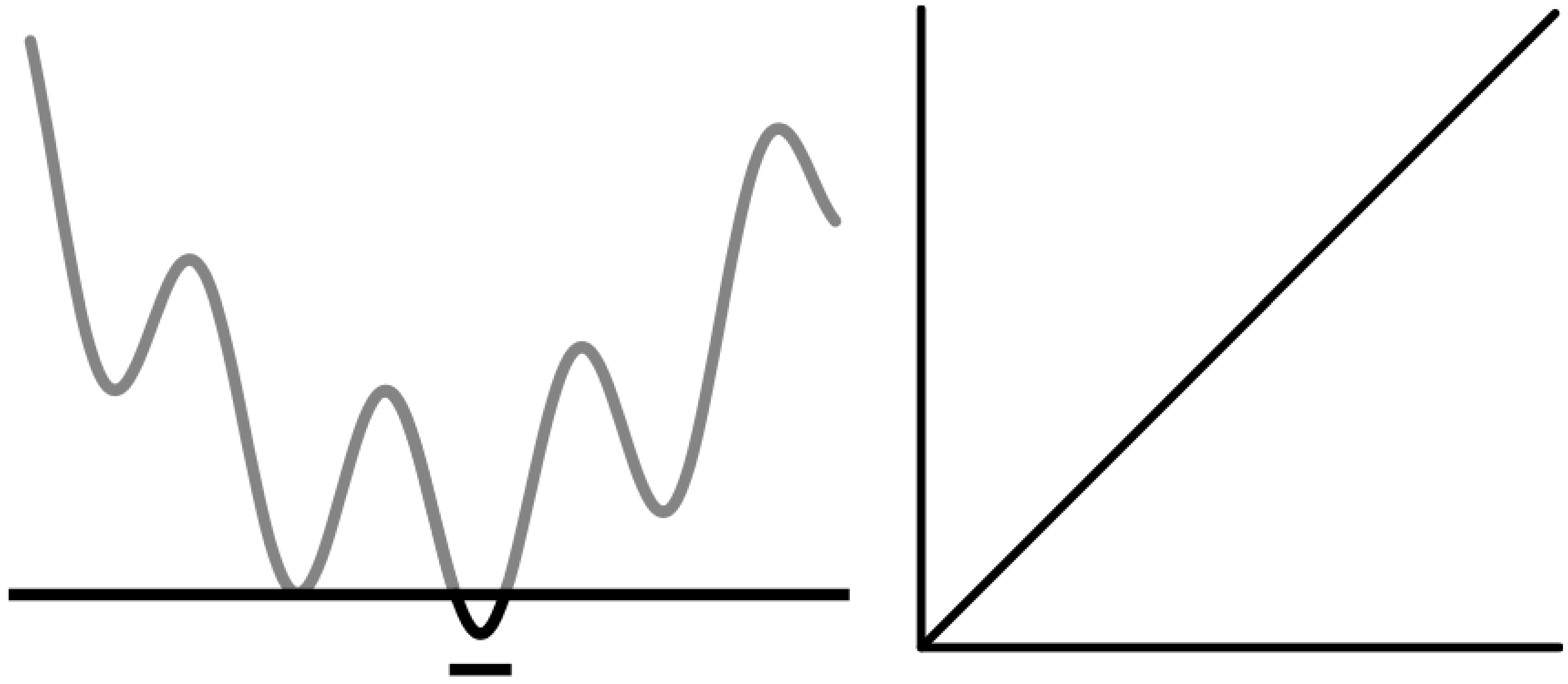

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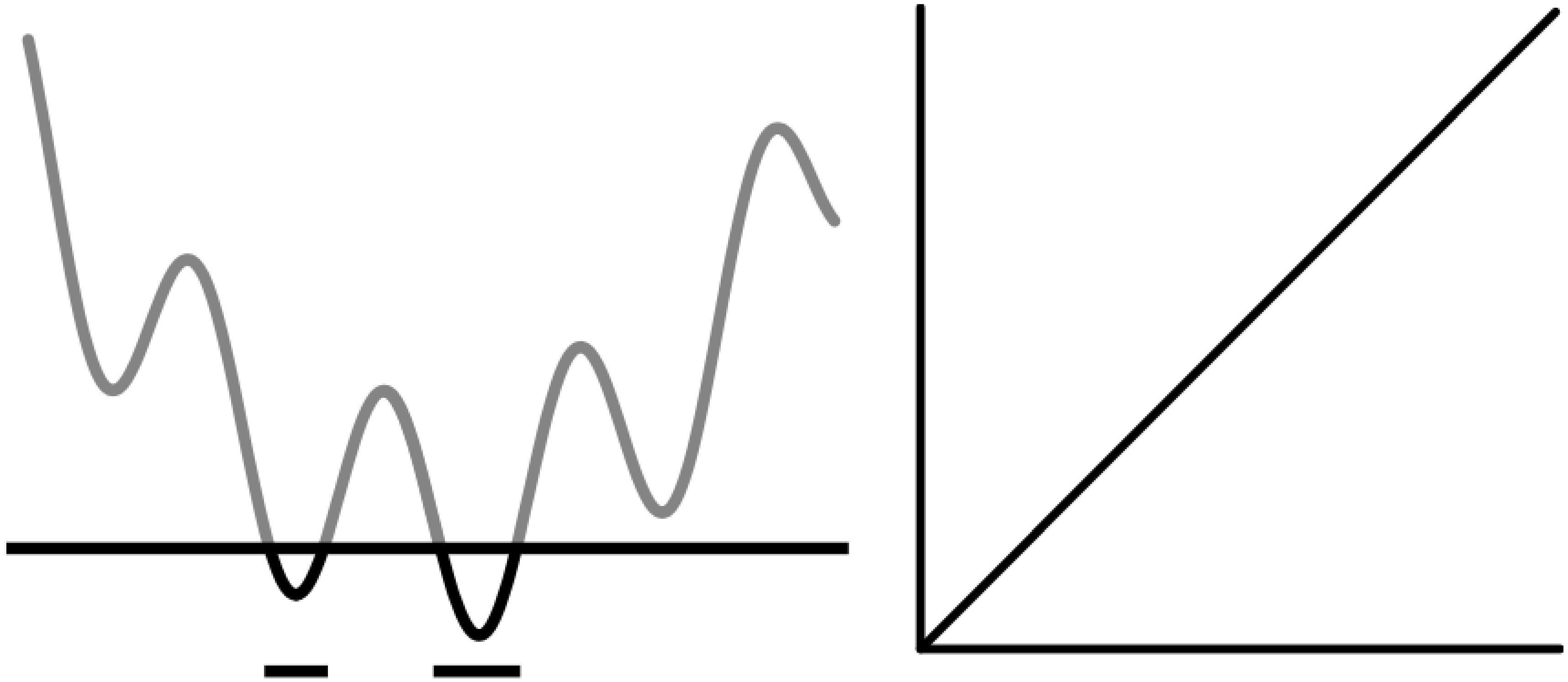

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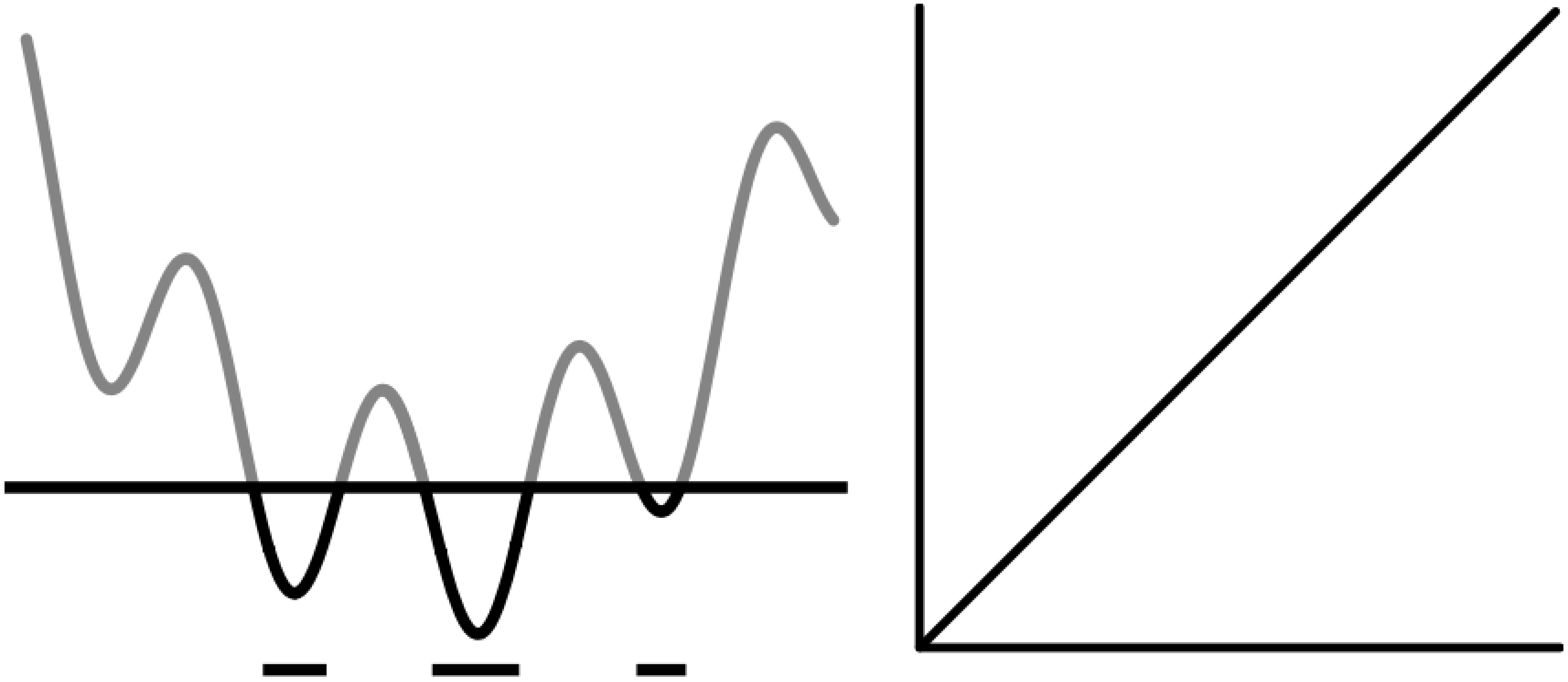

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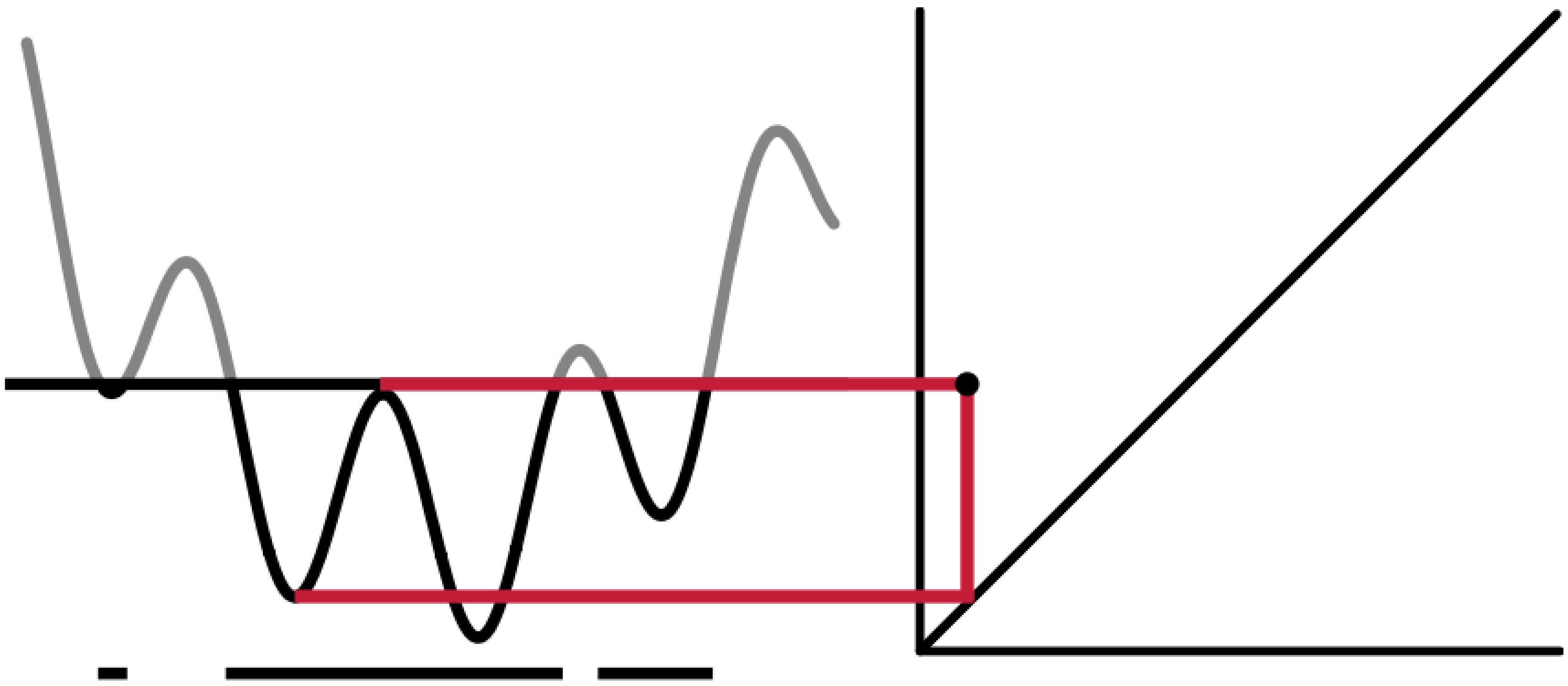

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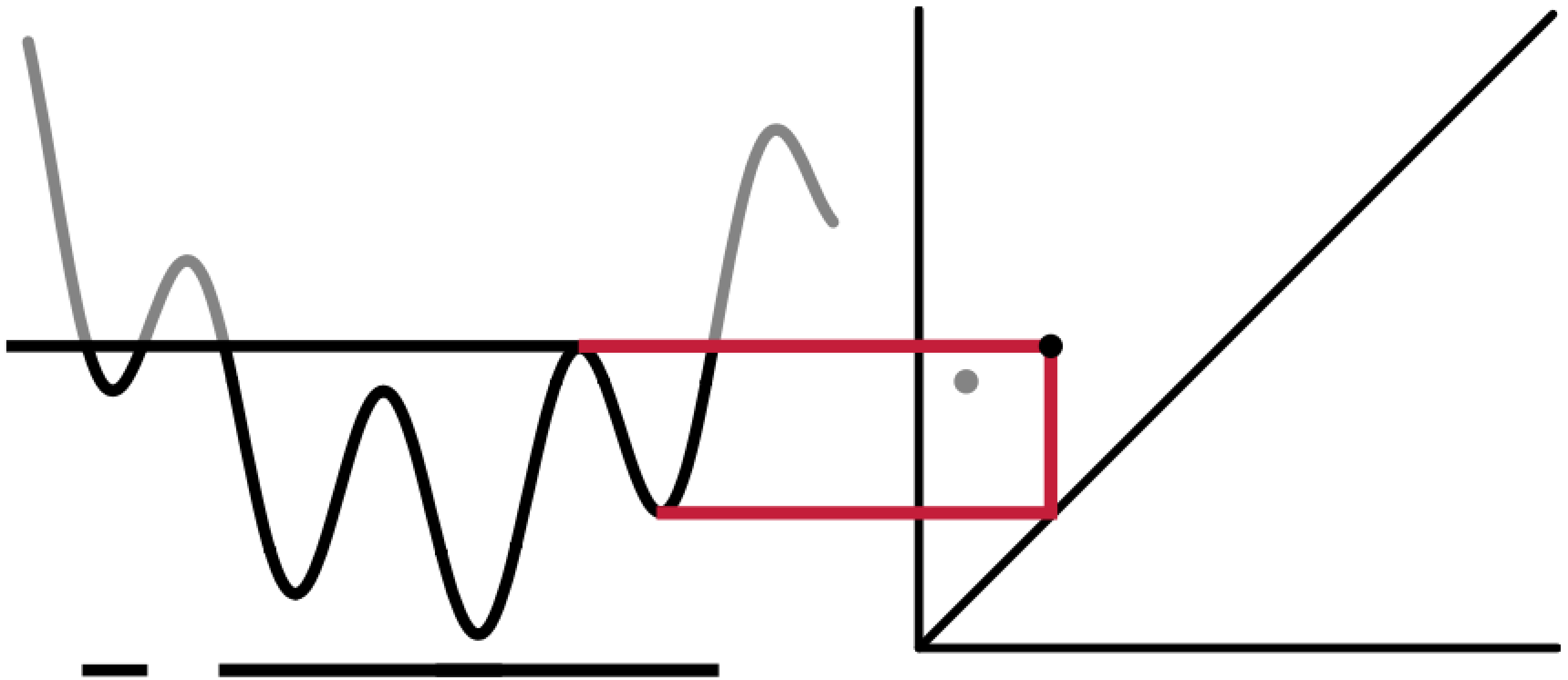

Persistence Diagram of a scalar function

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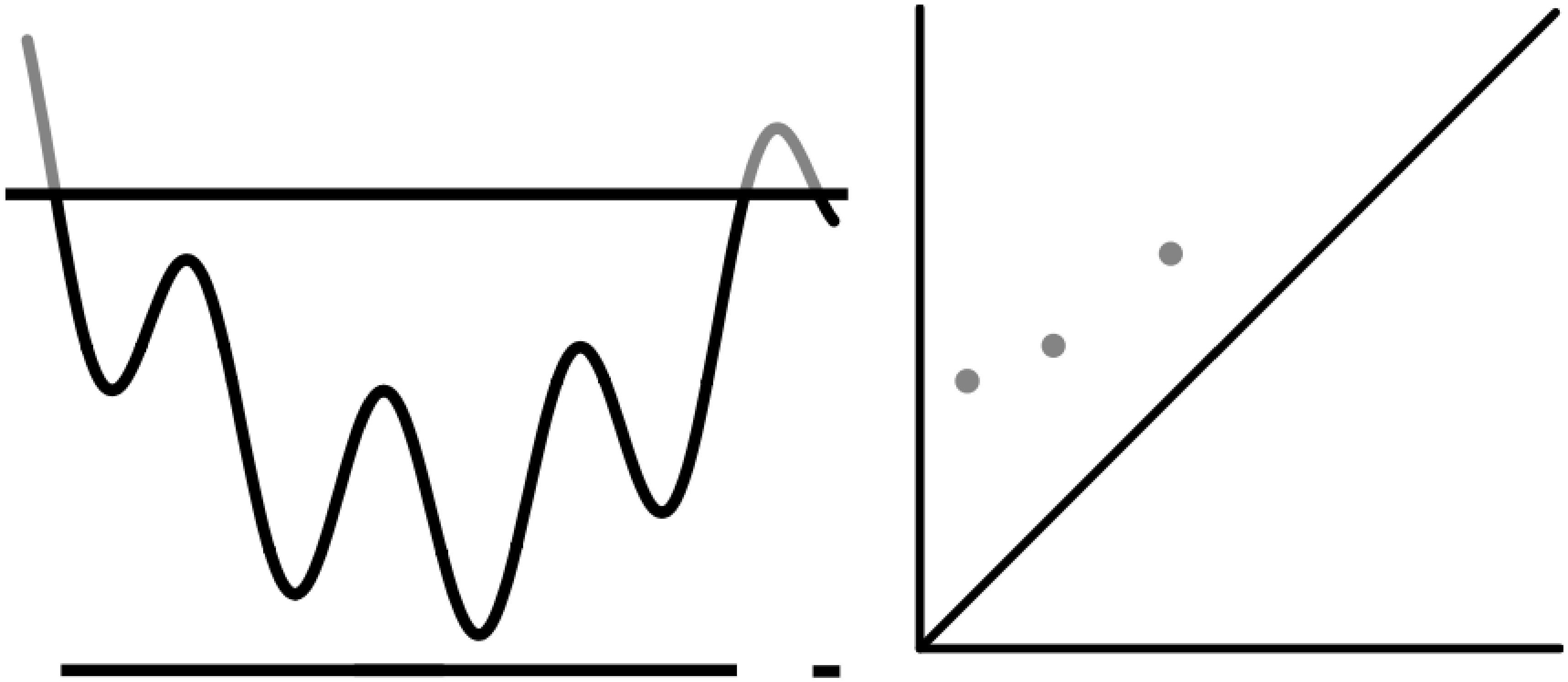

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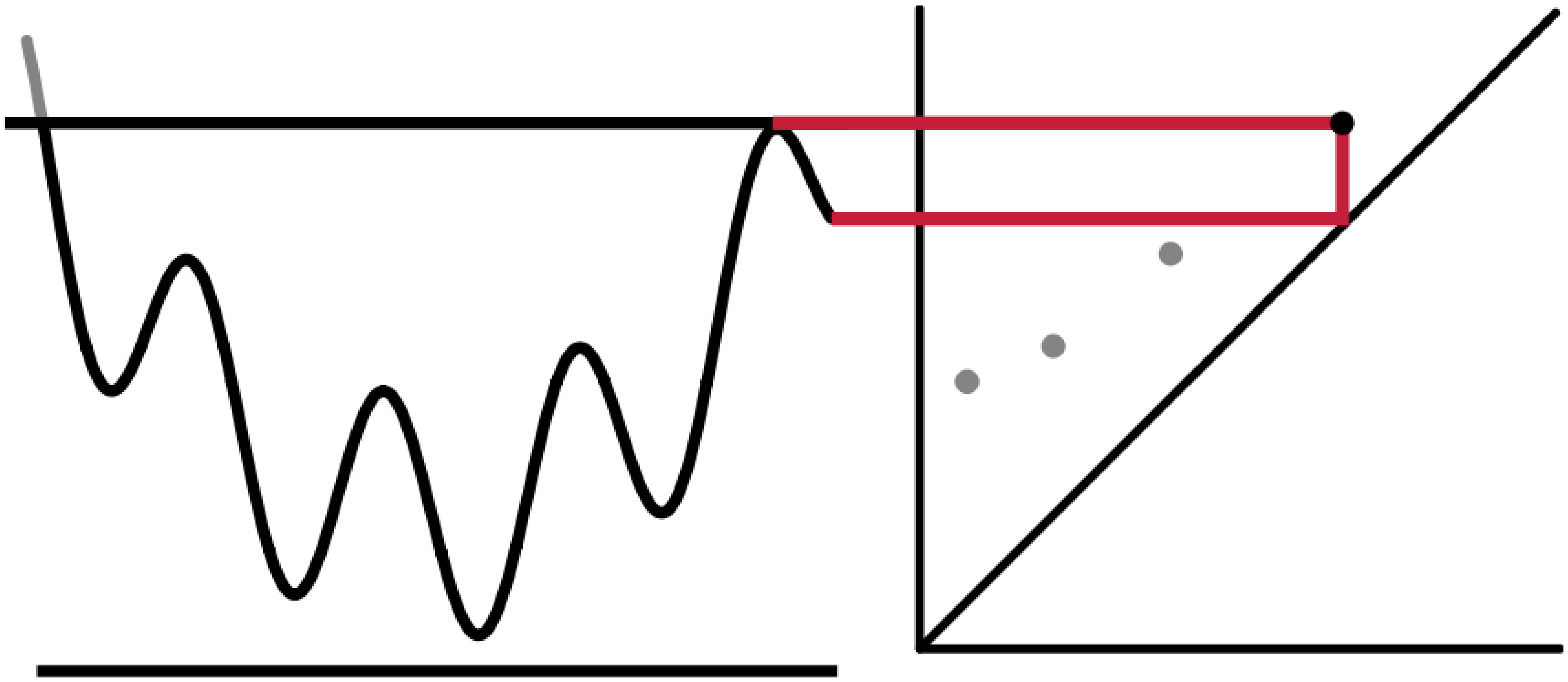

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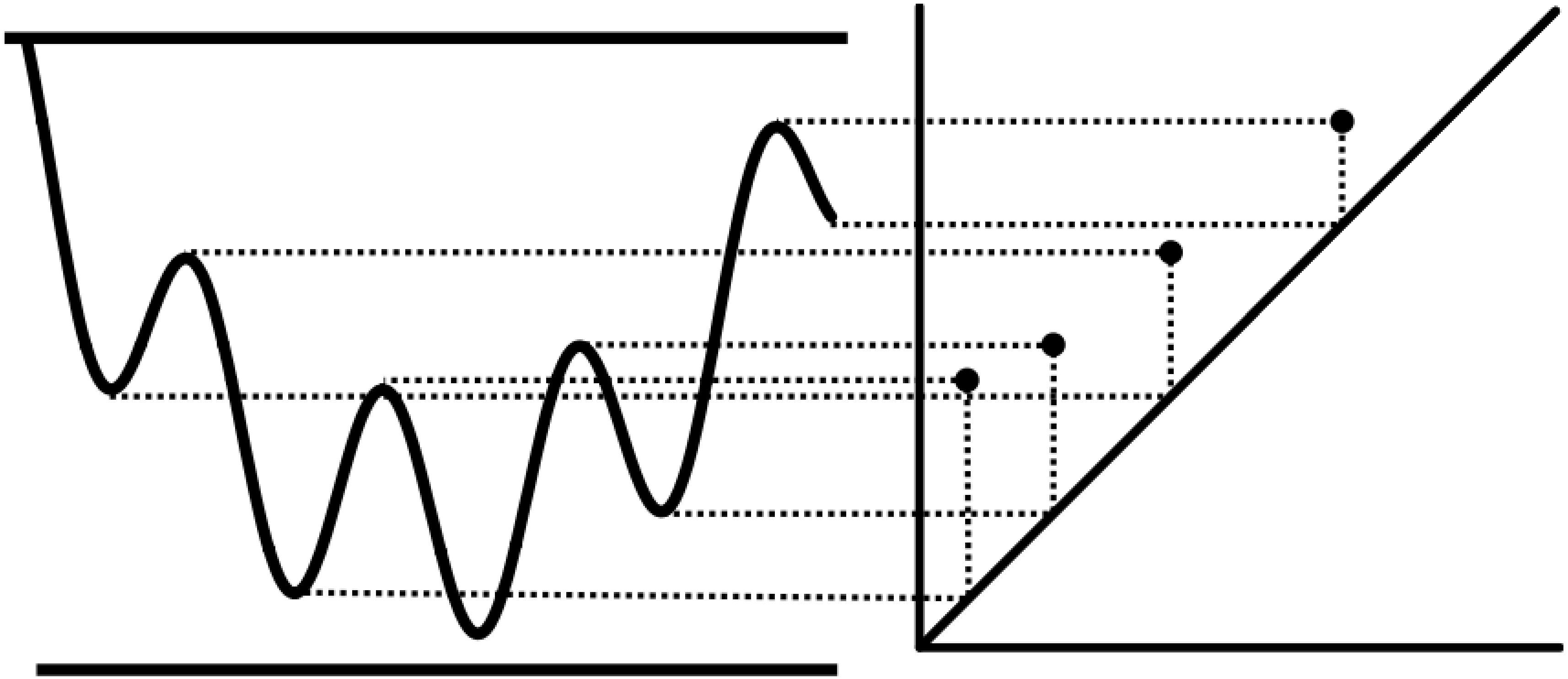
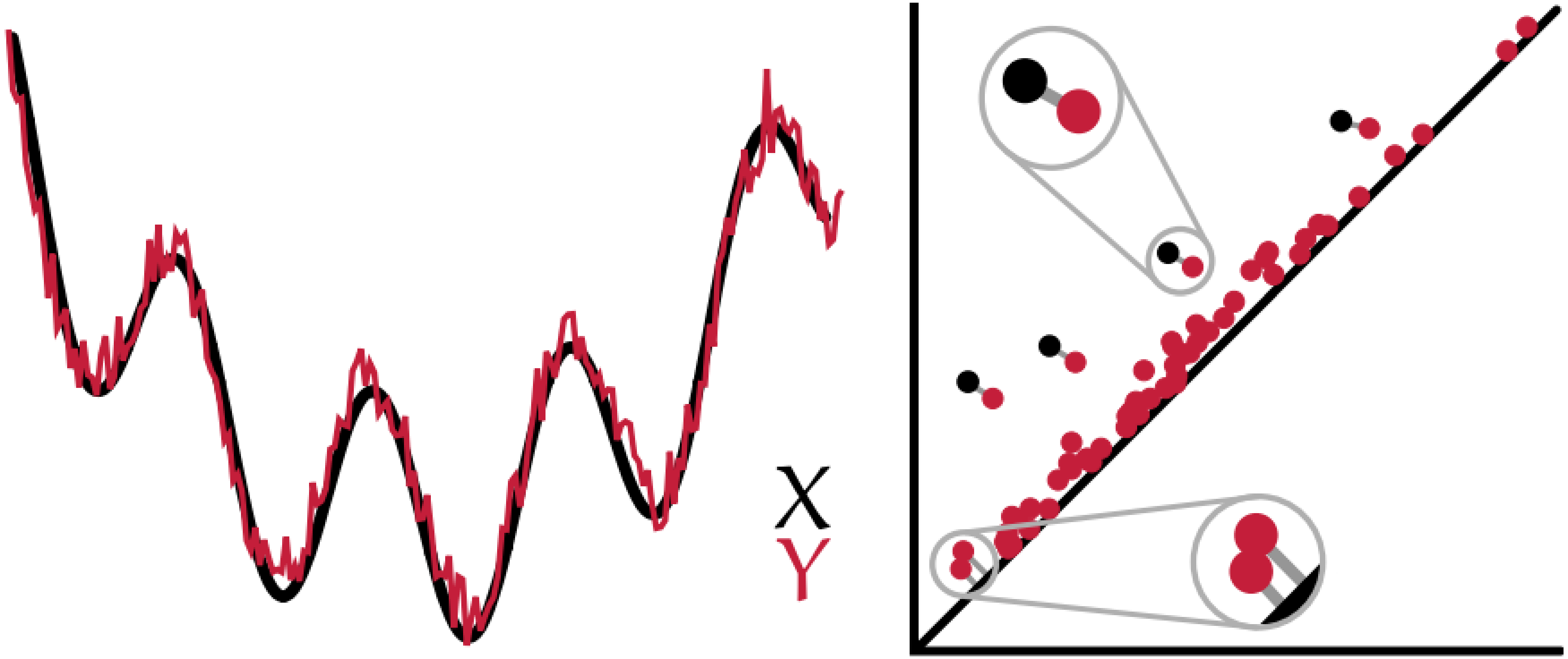


Another example



Persistence Diagram of a scalar function

Algorithm 2: Calculating 0-dimensional persistent homology

Require: A function $f\colon \mathbb{D} \subseteq \mathbb{R} \to \mathbb{R}$

function PersistentHomology(f)
    $U \leftarrow \varnothing$ ▷ Initialize an empty union–find structure
    Sort the function values of f in ascending order.
    for Function value y of f **do**
        if y is a local minimum **then**
            Create a new connected component in U.
        else if y is a local maximum **then**
            Use U to merge the two connected components meeting at y.
        else
            Use U to add y to the current connected component.
        end if
    end for
end function

Persistence Diagram of a scalar function

Algorithm 3: Calculating discrete 0-dimensional persistent homology

Require: A discrete sample $\{(x_1, y_1), (x_2, y_2), \dots\}$ of a function $f\colon \mathbb{D} \subseteq \mathbb{R} \to \mathbb{R}$

```
function PersistentHomology(f)
    U ← ∅                                          ▷ Initialize an empty union–find structure
    Sort the value tuples in ascending order, such that y_1 ≥ y_2 ≥ ...
    for Tuple (x_i, y_i) of f do
        if y_{i-1} > y_i and y_{i+1} > y_i then                     ▷ y_i is a local minimum
            U.add(i)                           ▷ Create a new connected component in U
        else if y_{i-1} < y_i and y_{i+1} < y_i then                ▷ y_i is a local maximum
            c ← U.get(i − 1)                                ▷ Get first connected component
            d ← U.get(i + 1)                               ▷ Get second connected component
            U.merge(c, d)             ▷ Merge the two connected components meeting at y_i
        else                                                          ▷ y_i is a regular point
            c ← U.get(i − 1)                                      ▷ Get connected component
            U[c] ← U[c] ∪ i                    ▷ Add y_i to the current connected component
        end if
    end for
    return U
end function
```