Nearest Neighbors-Based Clustering and Classification Methods

Mustafa Hajij

Application of K-nearest neighbors: $K\ Neighbors\ Classifier$

This is an supervised classifier that takes into consideration the nearest k-points to an input point to determine the class that point belongs to. The idea is to use the *majority vote* of the neighbors of the input points.

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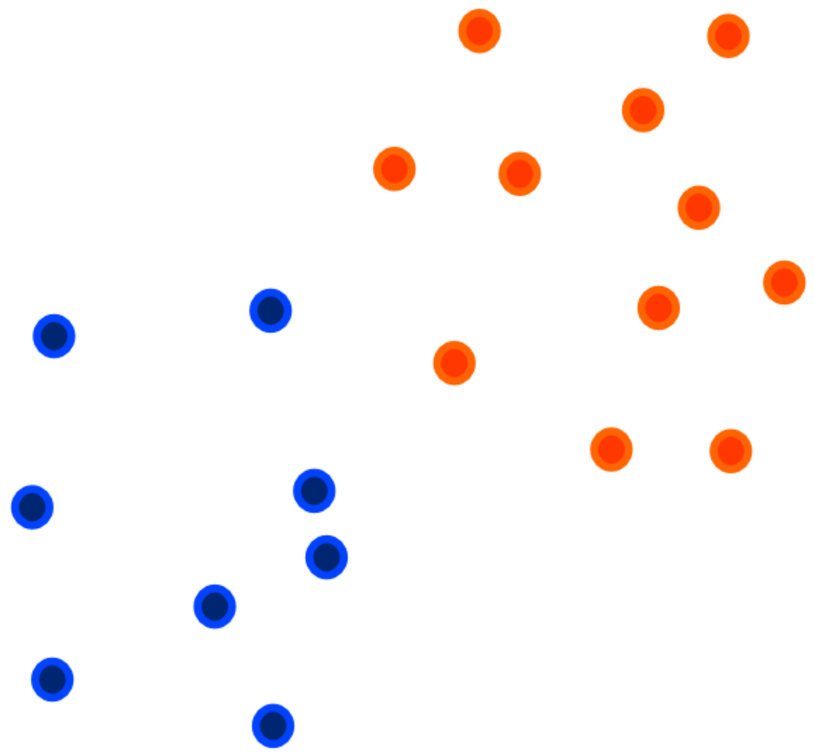

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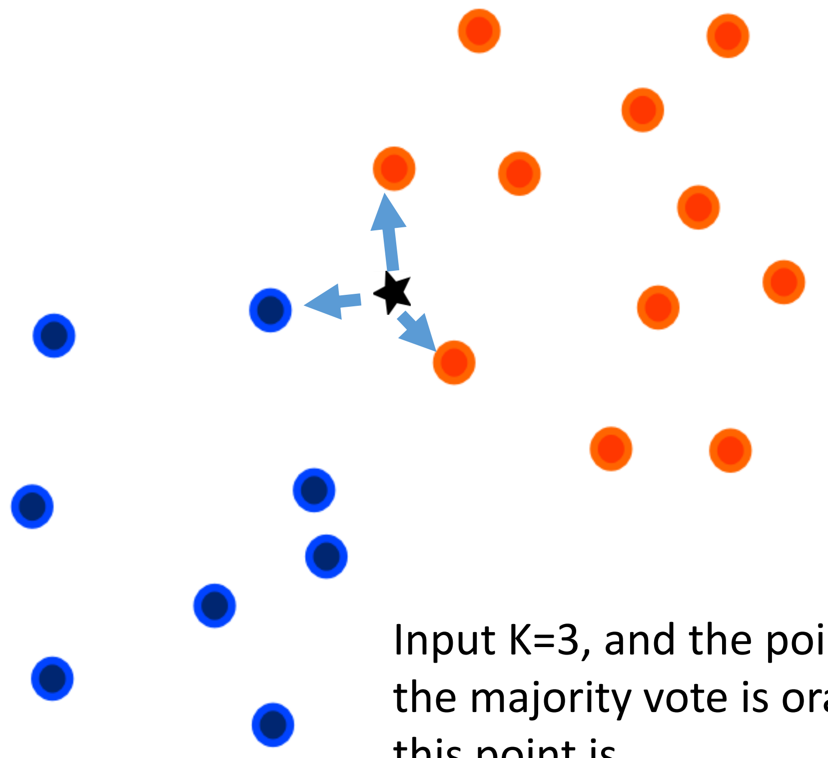

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the majority vote is orange hence
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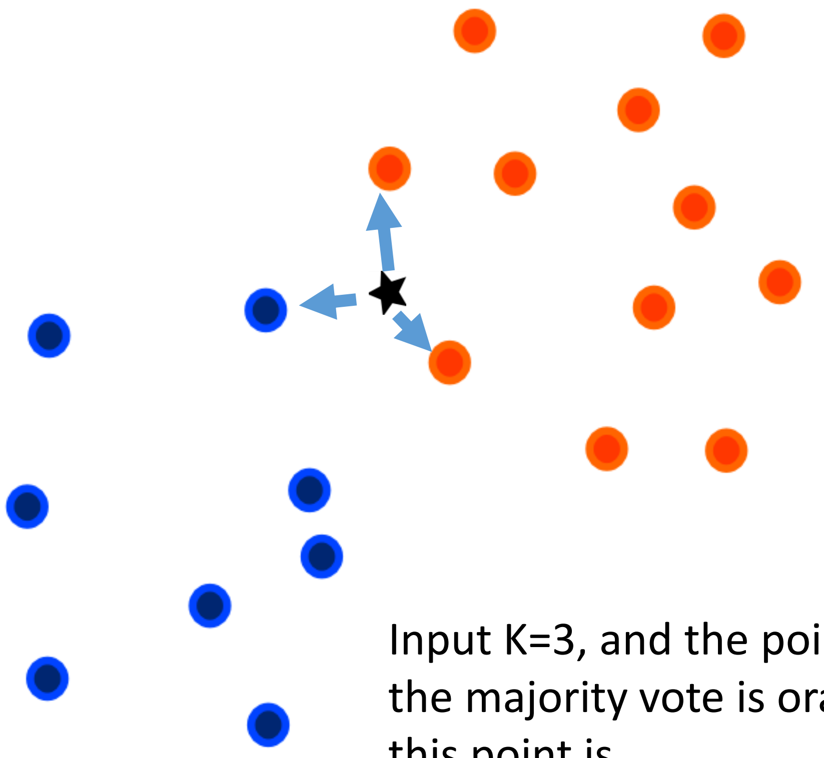

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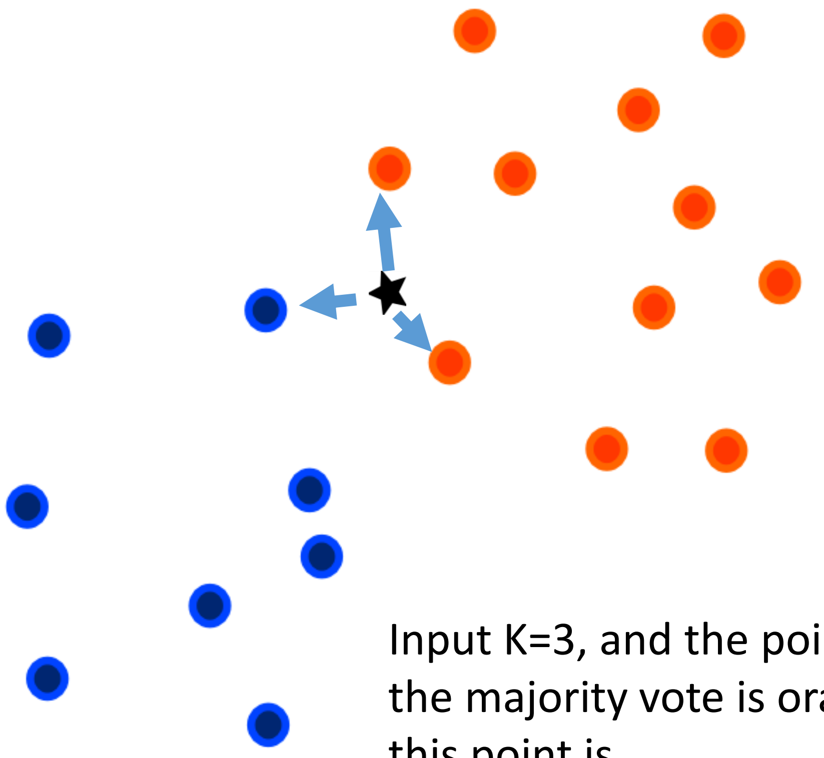

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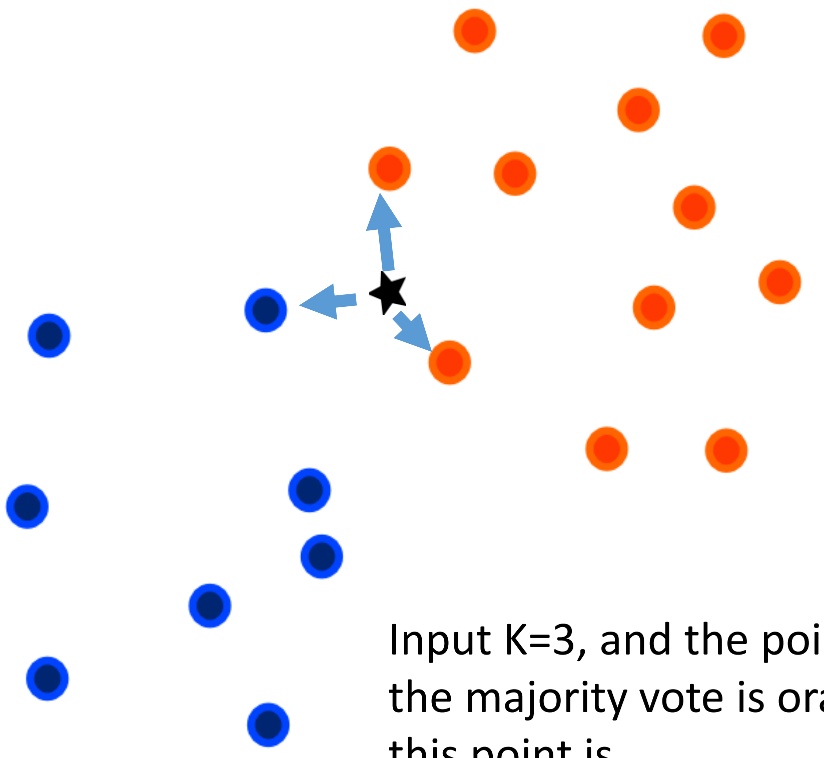

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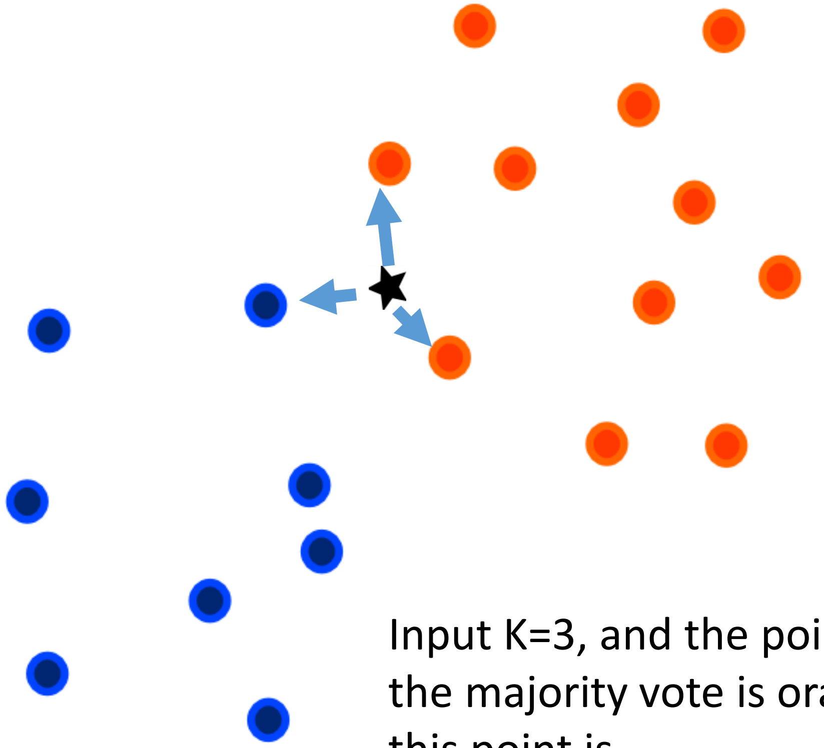

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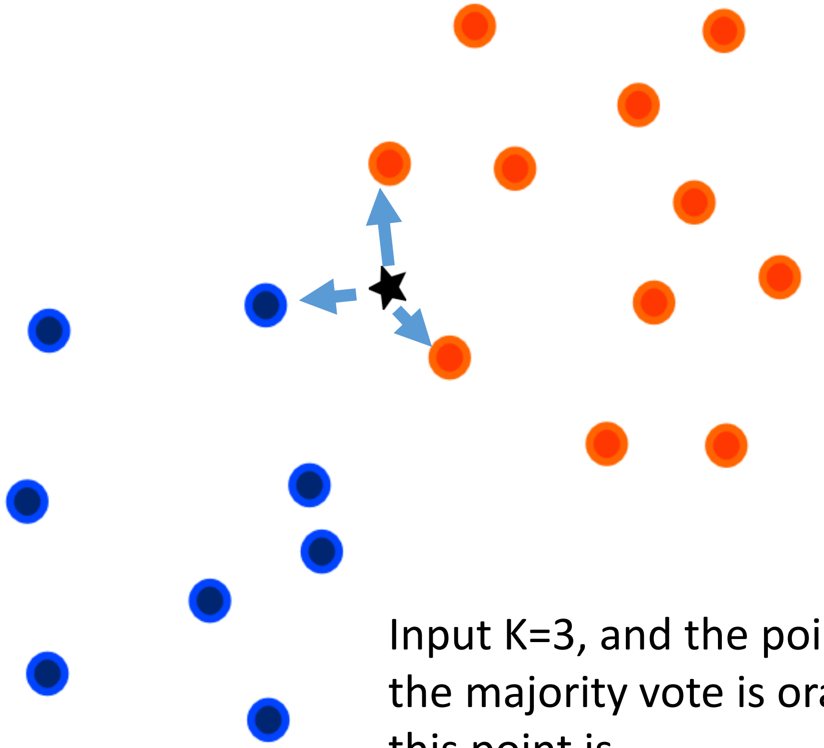

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See this sklearn example

Application of ε-nearest neighbors: *Radius Neighbors Classifier*

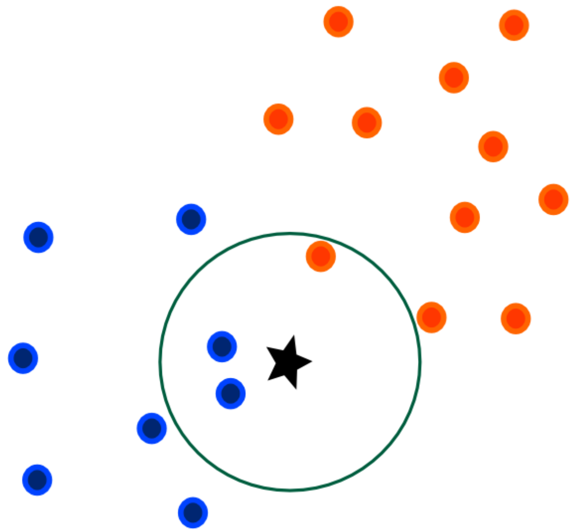
The idea is really similar to before, here instead of consider the closest k-nearest neighbor to determine the class, we Consider all instances within radius ε to determine the class of the input point

In sklearn :

```
from sklearn.neighbors import RadiusNeighborsClassifier

X = [[0], [1], [2], [3]]
y = [0, 0, 1, 1]

neigh = RadiusNeighborsClassifier(radius=1.0)
neigh.fit(X, y)                              # fit the data
```



Sklearn [example]

Neighborhood graphs and their relatives (review)

K-Nearest Neighbor Graph (KNN Graph)

Suppose that we are given a set of points $X = \{p_1, p_2, \dots, p_n\}$ in R^d with a distance function d defined one them.

Sklearn implementation

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For a fixed integer k, connect the points x, y in X if either $d(x,y) \leq d(x, x_k)$ or $d(x,y) \leq d(y, y_k)$ where x_k, y_k are the k^{th} nearest neighbors of x, y respectively. Doing do for all points we obtain a graph $G(X, E)$ where E is the set of edges connect the points in X as described above.

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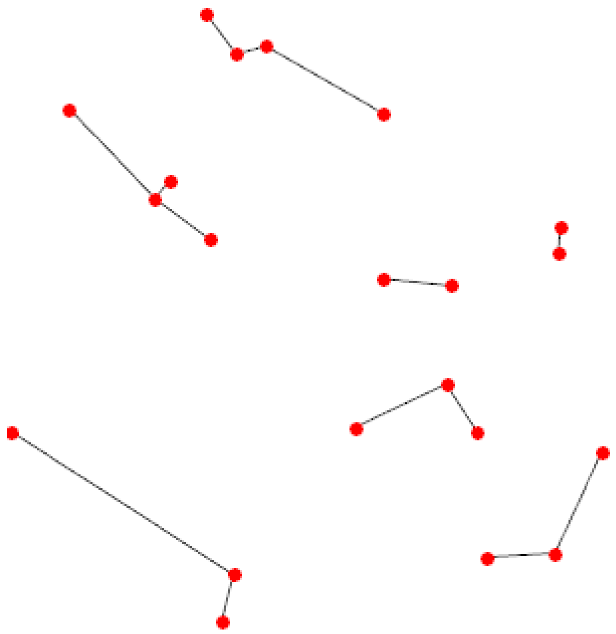

Example of 1-NN graph

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The graph that we obtain is called the K-nearest neighbor graph or K-NN graph.

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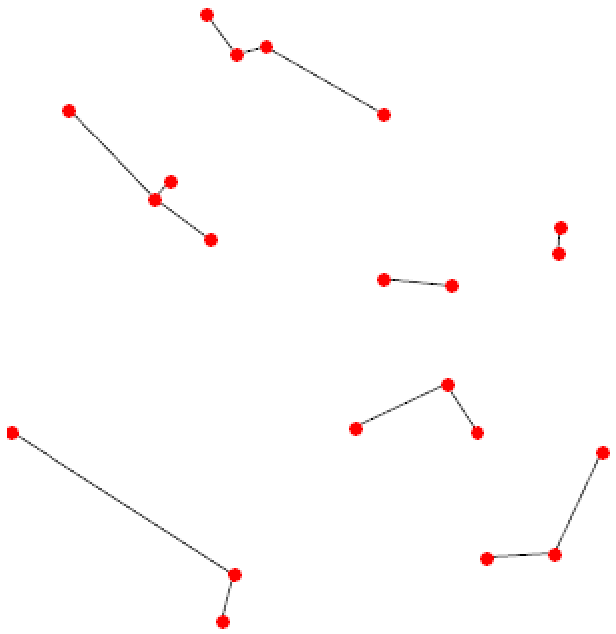

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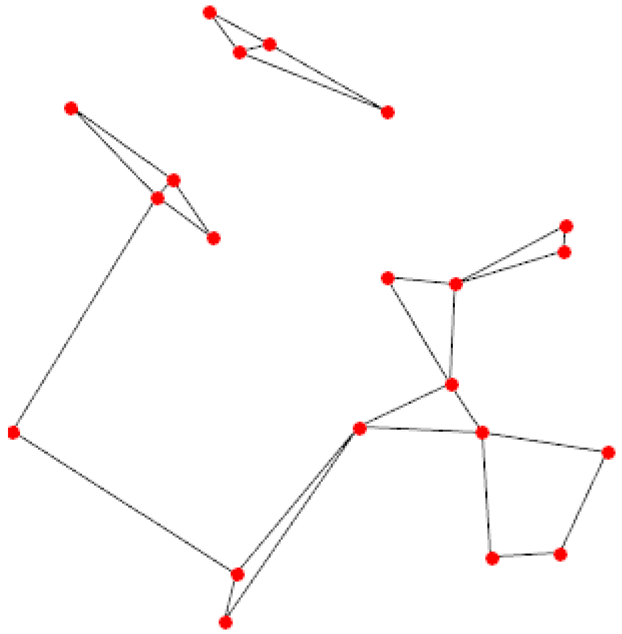

Example of 2-NN graph

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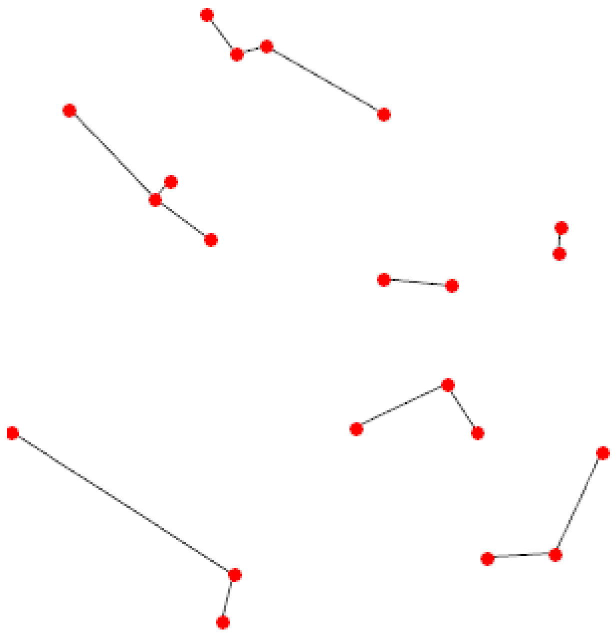

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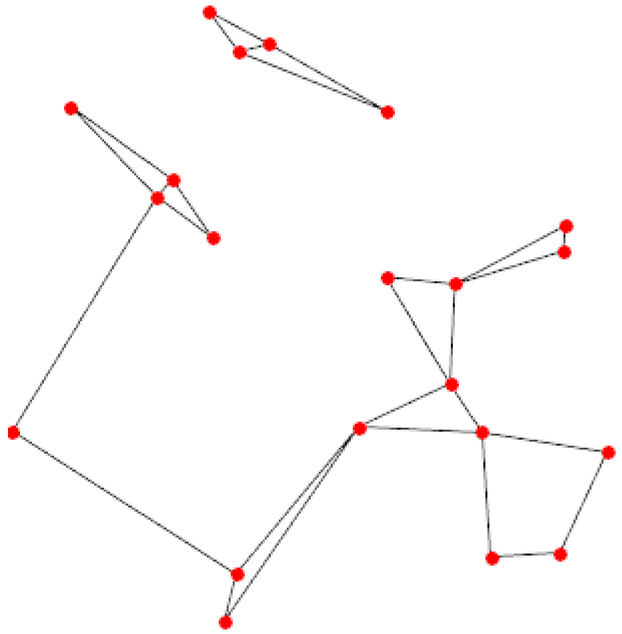
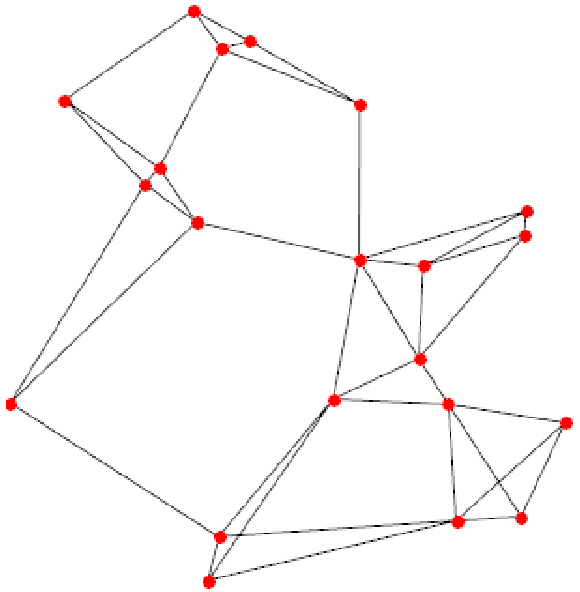


Example of 2-NN graph



Example of 3-NN graph

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ε- neighborhood graph

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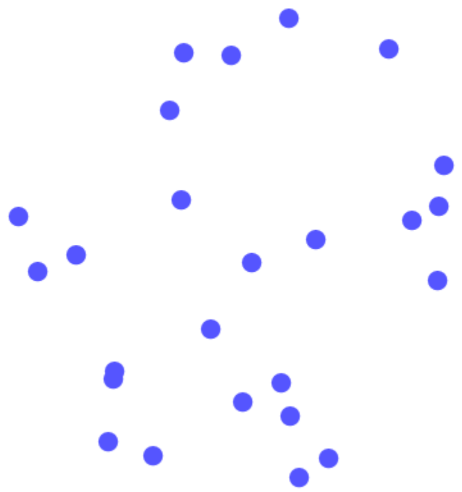

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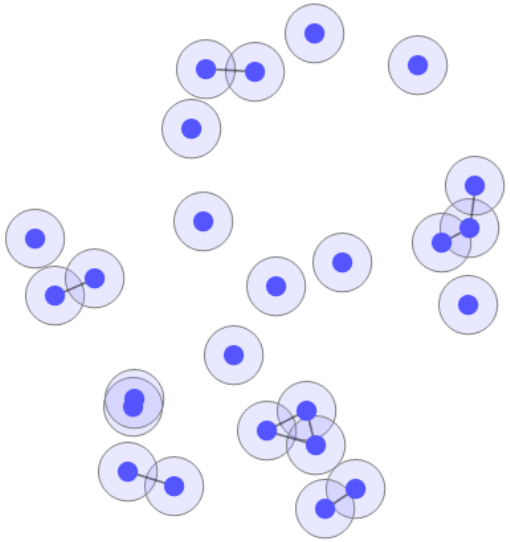
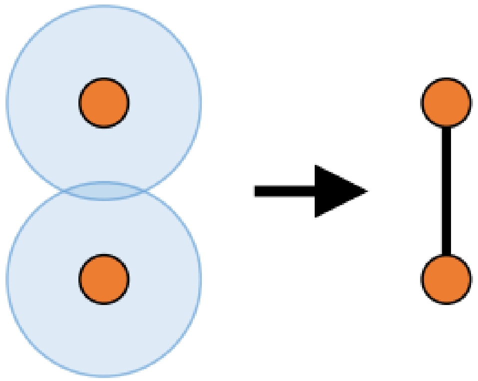
This is *equivalent* to :
Insert an edge if the disks around the points intersect each other.





Note that $d(x, y) \leq \frac{\varepsilon}{2}$ if and only if the two balls surrounding x and y intersect

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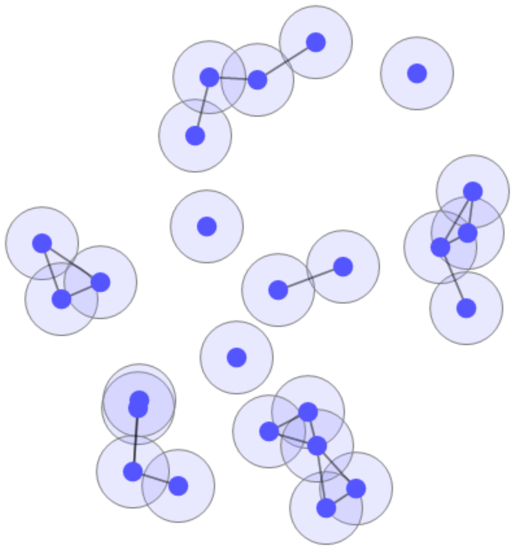

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Sklearn implementation

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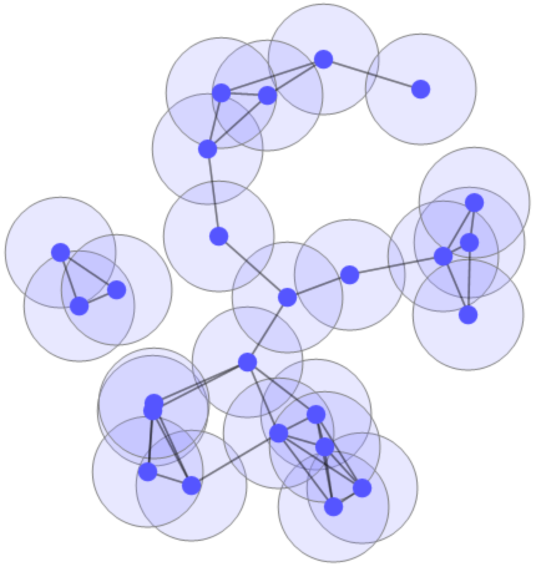

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Sklearn implementation

Euclidian Minimal Spanning Tree (EMST)

Suppose that we are given a set of points $X = \{p_1, p_2, \ldots, p_n\}$ in R^d with a distance function d defined one them.

The EMST of X is a minimal spanning tree where the weight of the edge between each pair of points is the distance between those two points.

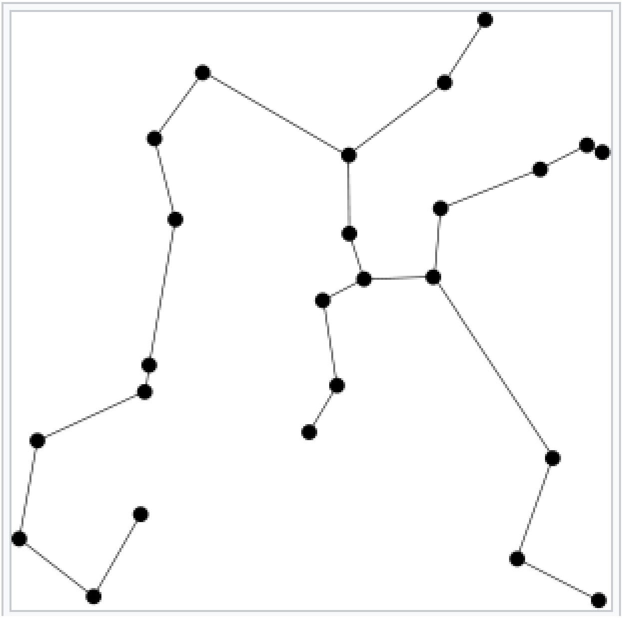

Image source-Wikipedia article

Graph Based Clustering Algorithms : Zahn's algorithm

Suppose that we are given a set of points $X = \{p_1, p_2, \ldots, p_n\}$ in R^d with a distance function d defined one it.

1. Construct the EMST of X.
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This definition of consistent is not always ideal!. See here

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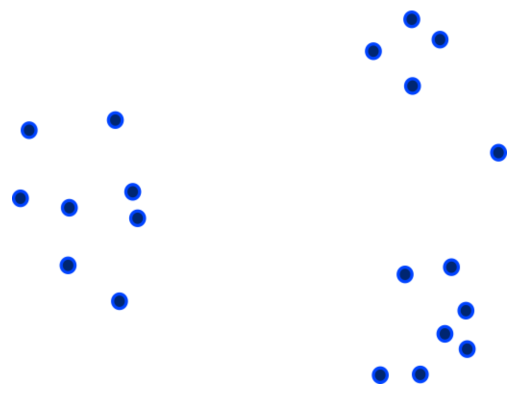

Input : point cloud and
Number of connected
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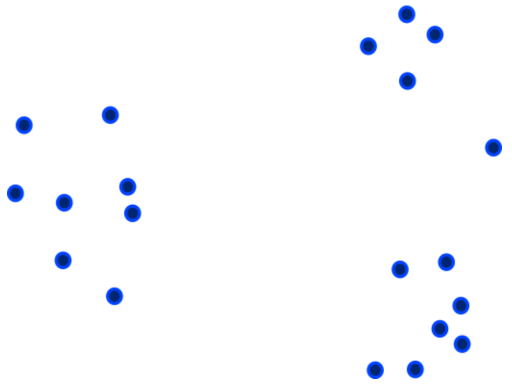

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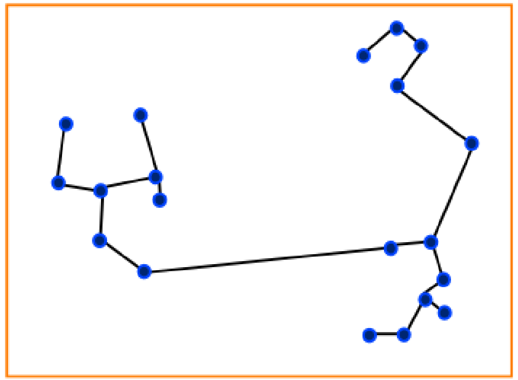

Construct EMST

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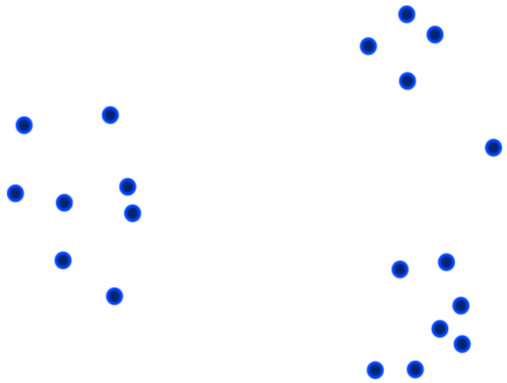

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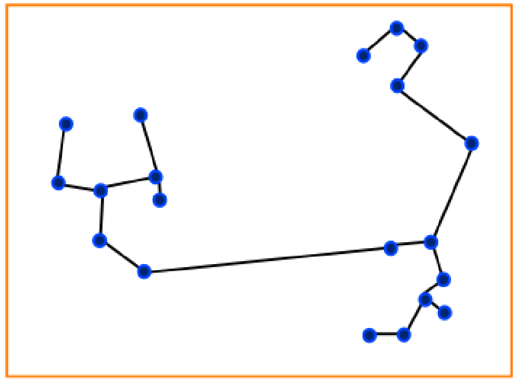

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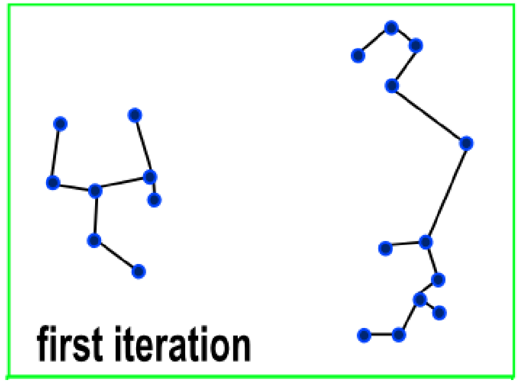

Delete an edge

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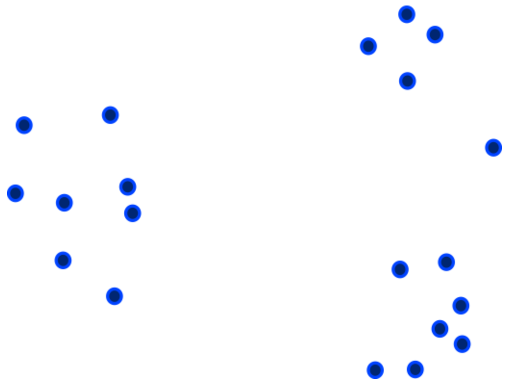

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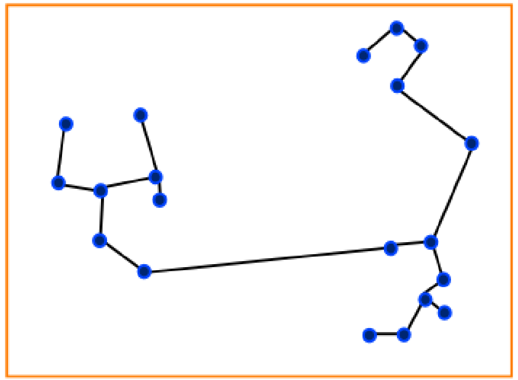

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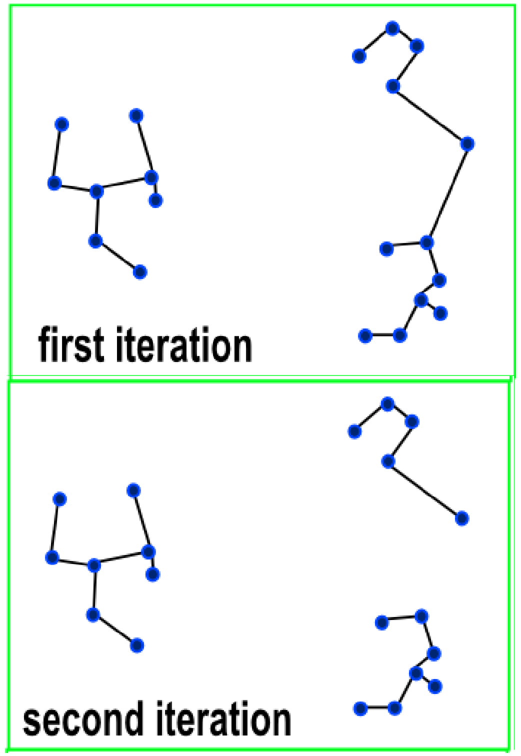

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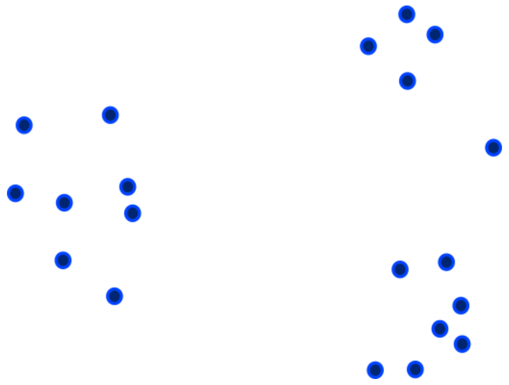

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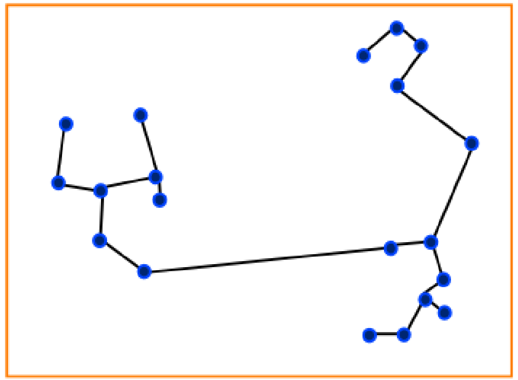

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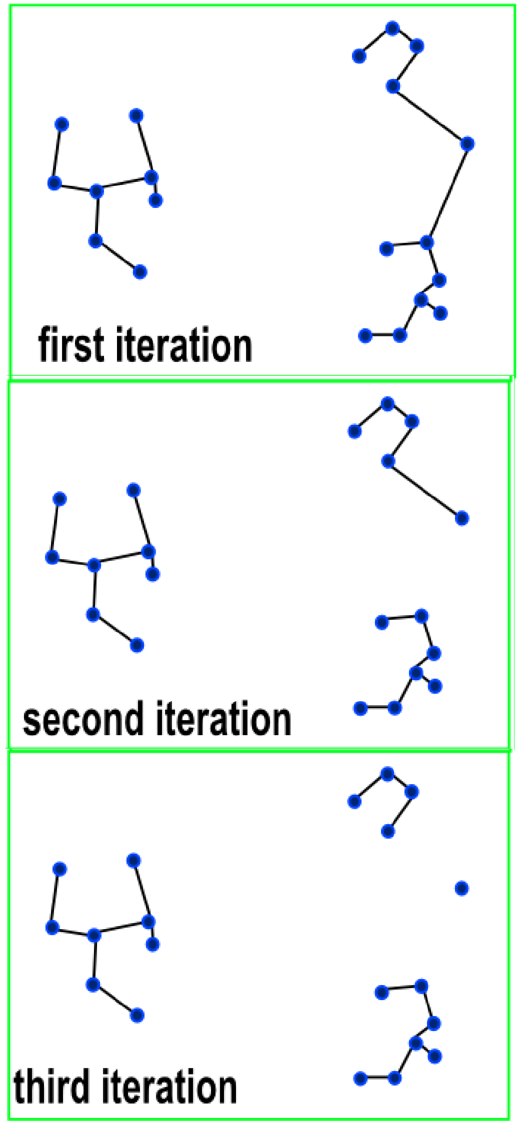

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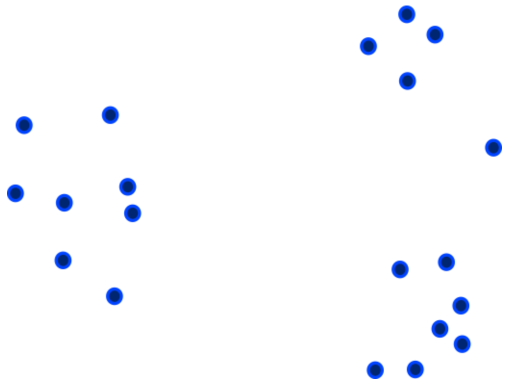

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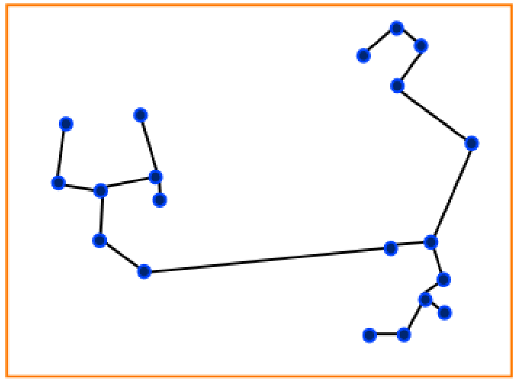

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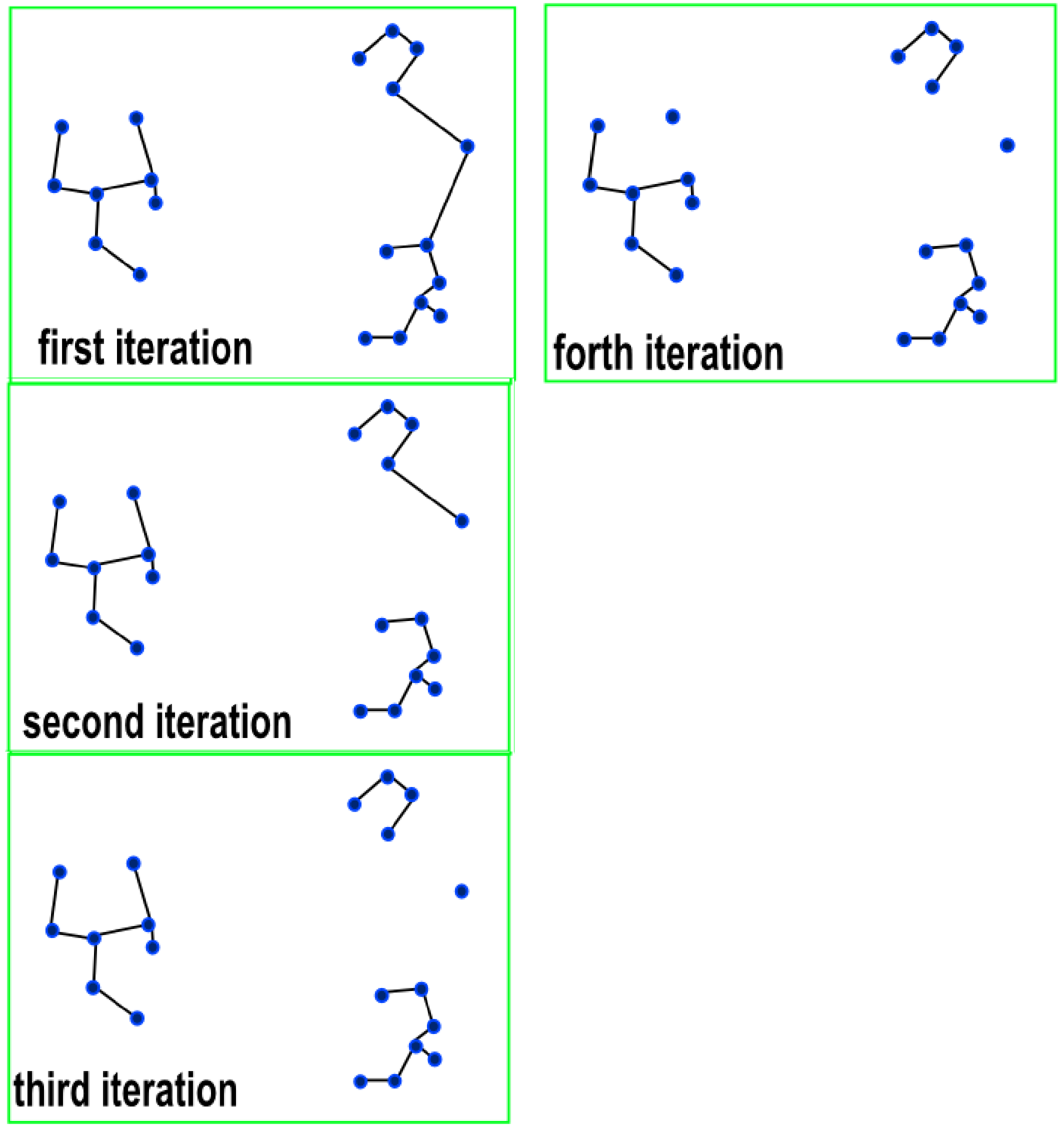
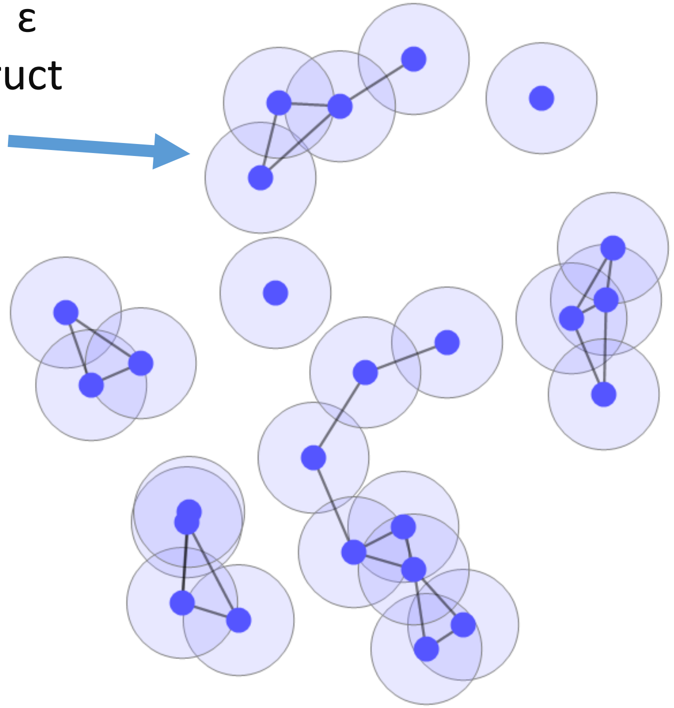


Until the termination criterion is satisfied

Graph Based Clustering Algorithms : ε- neighborhood graph

Choose an ε
And construct
the graph



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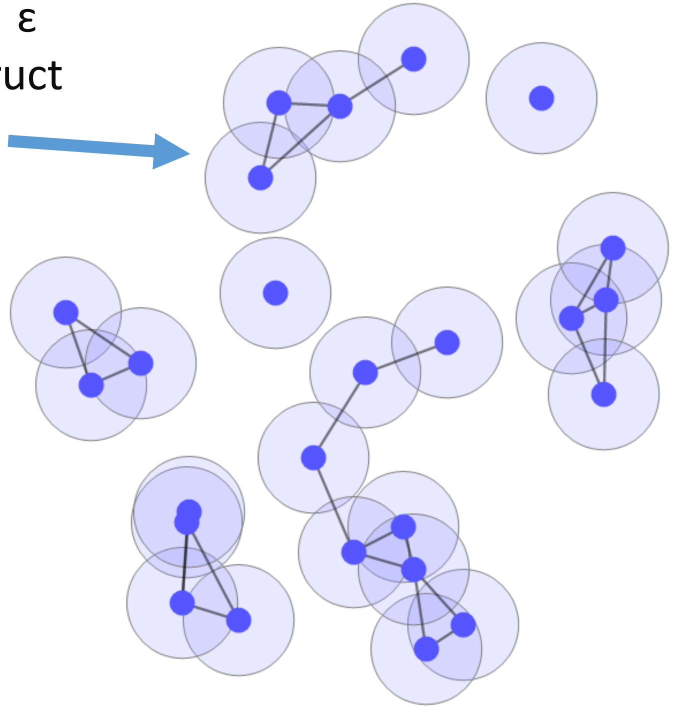
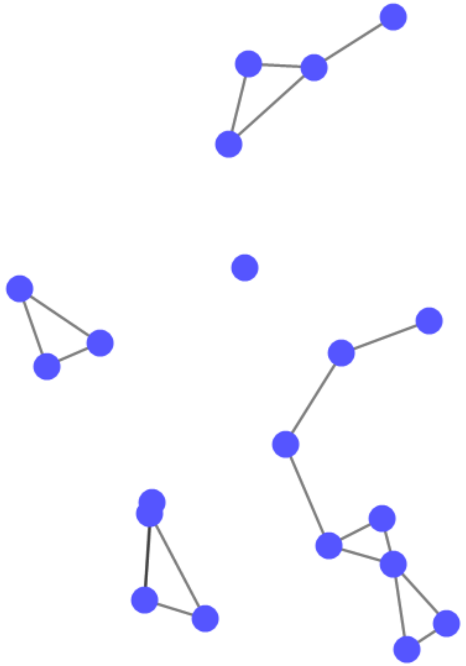


Consider the
connected
components



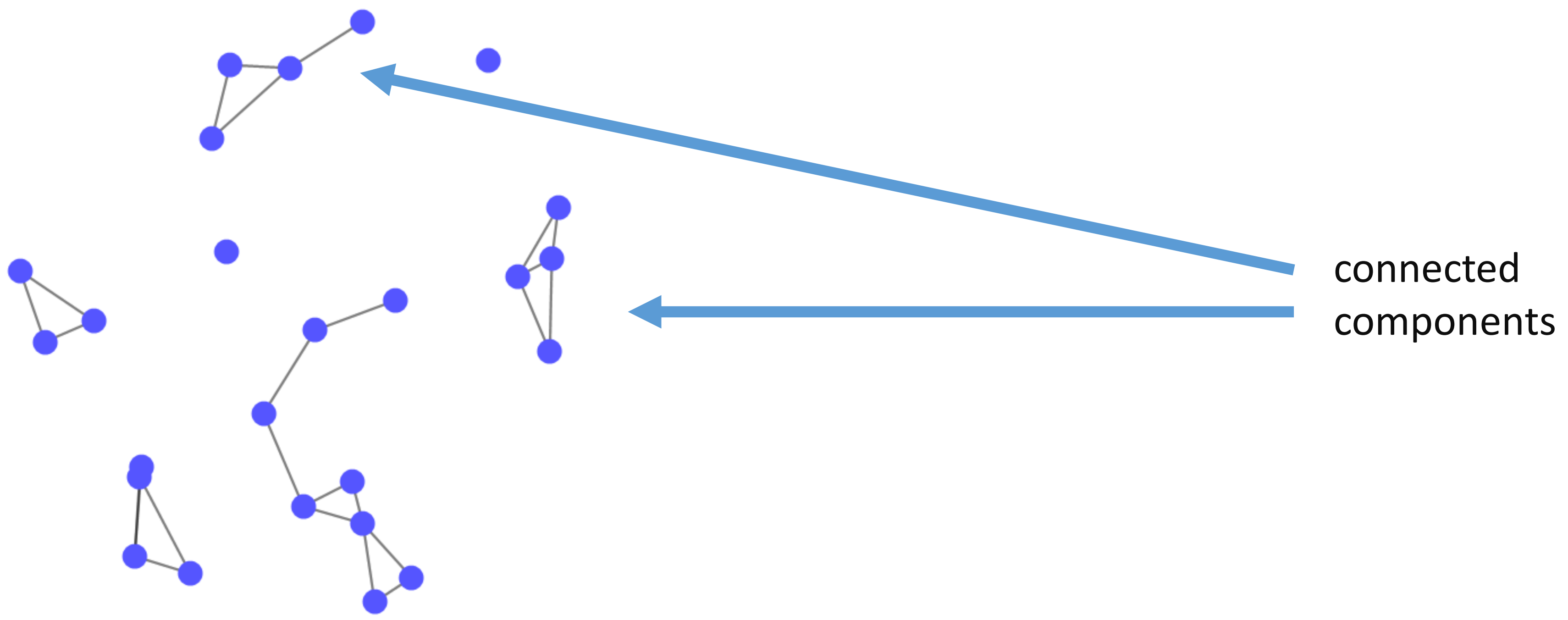
the points of
each connected
component form
a cluster

Recall : connected components of a graph

What is a connected component of a graph ?

How can we find the connected components of a graph ?

See this video for a review



Networkx

Introduction on networkx and how to use its basic functions.

```
import networkx as nx
G=nx.Graph()
G.add_edge(1,2)  # default edge data=1
G.add_edge(2,3,weight=0.9) # specify edge data
```

Finding connected components of a graph using networkx. See here

```
import networkx as nx
G = nx.path_graph(4)
G.add_path([10, 11, 12])
sorted(nx.connected_components(G), key = len, reverse=True)
```

Drawing a graph using networkx

```
import matplotlib.pyplot as plt
import networkx as nx
G=nx.path_graph(3)
nx.draw(G)  # networkx draw()
plt.draw()  # pyplot draw()
```