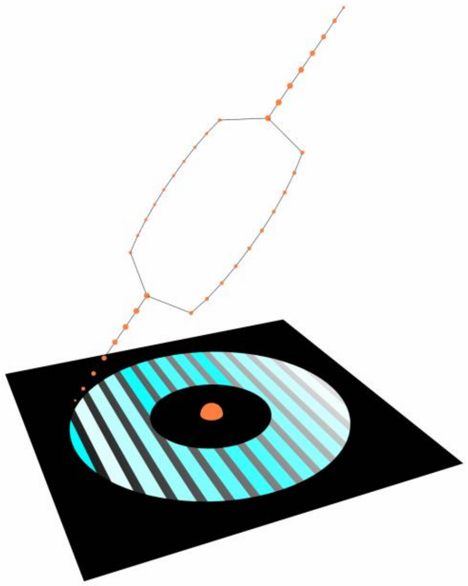
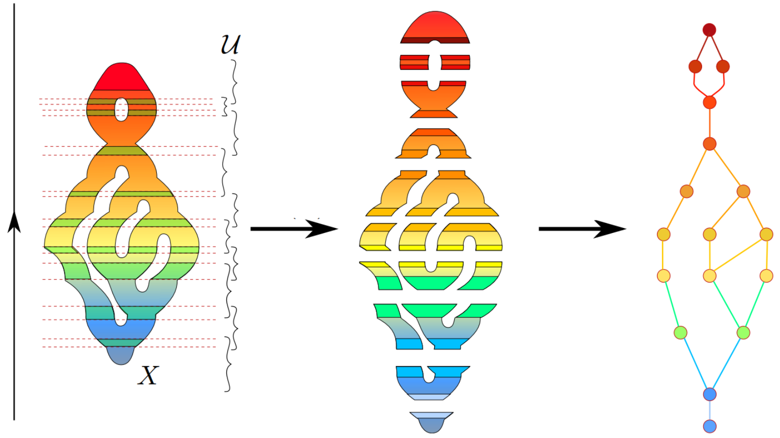
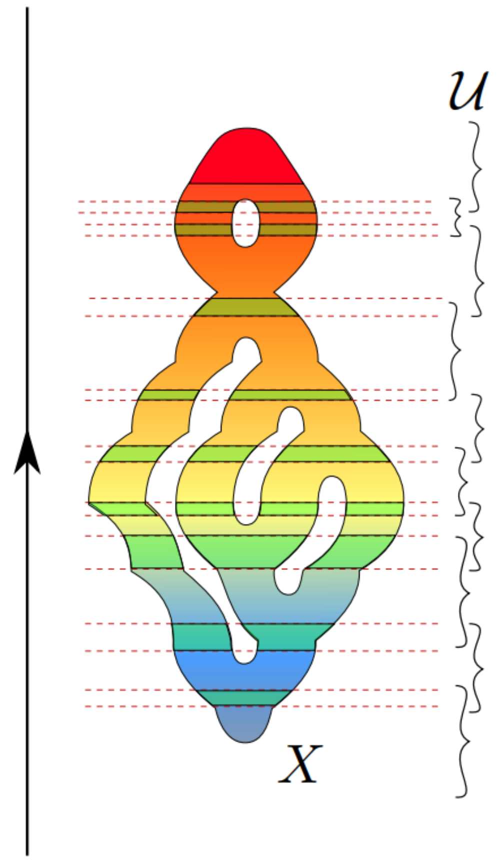


The Mapper Algorithm



Mustafa Hajij

Mapper Construction



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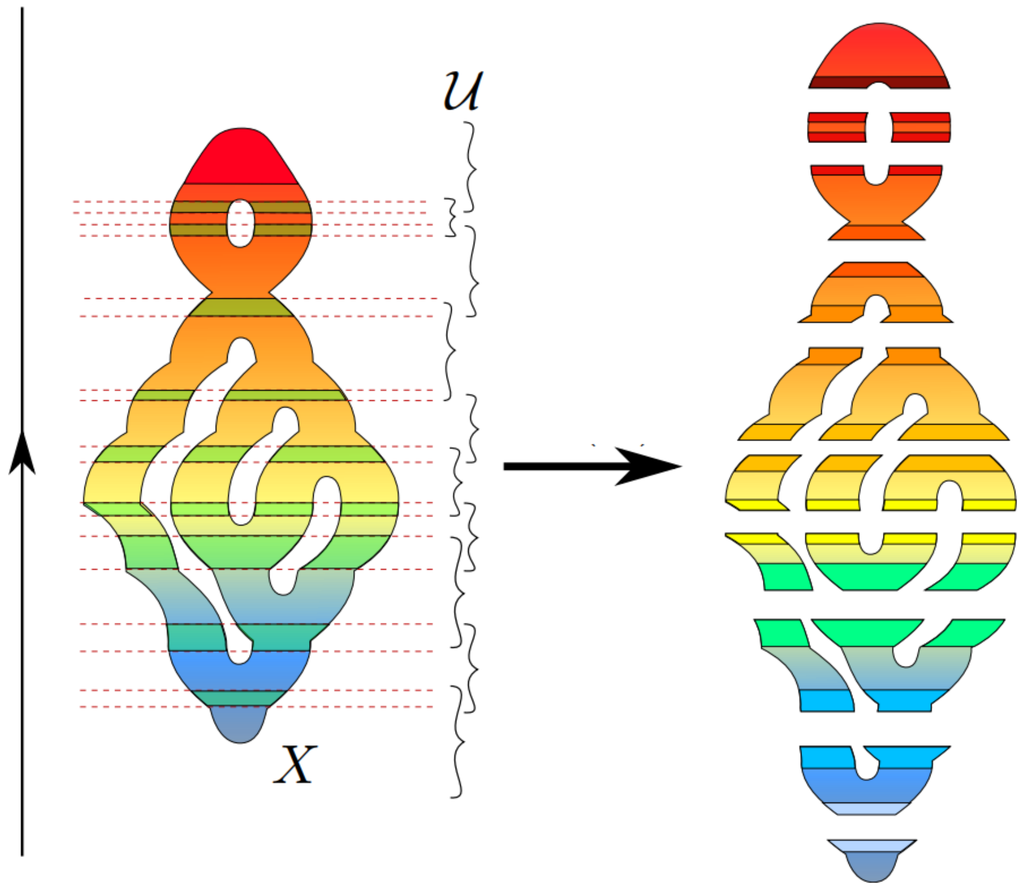

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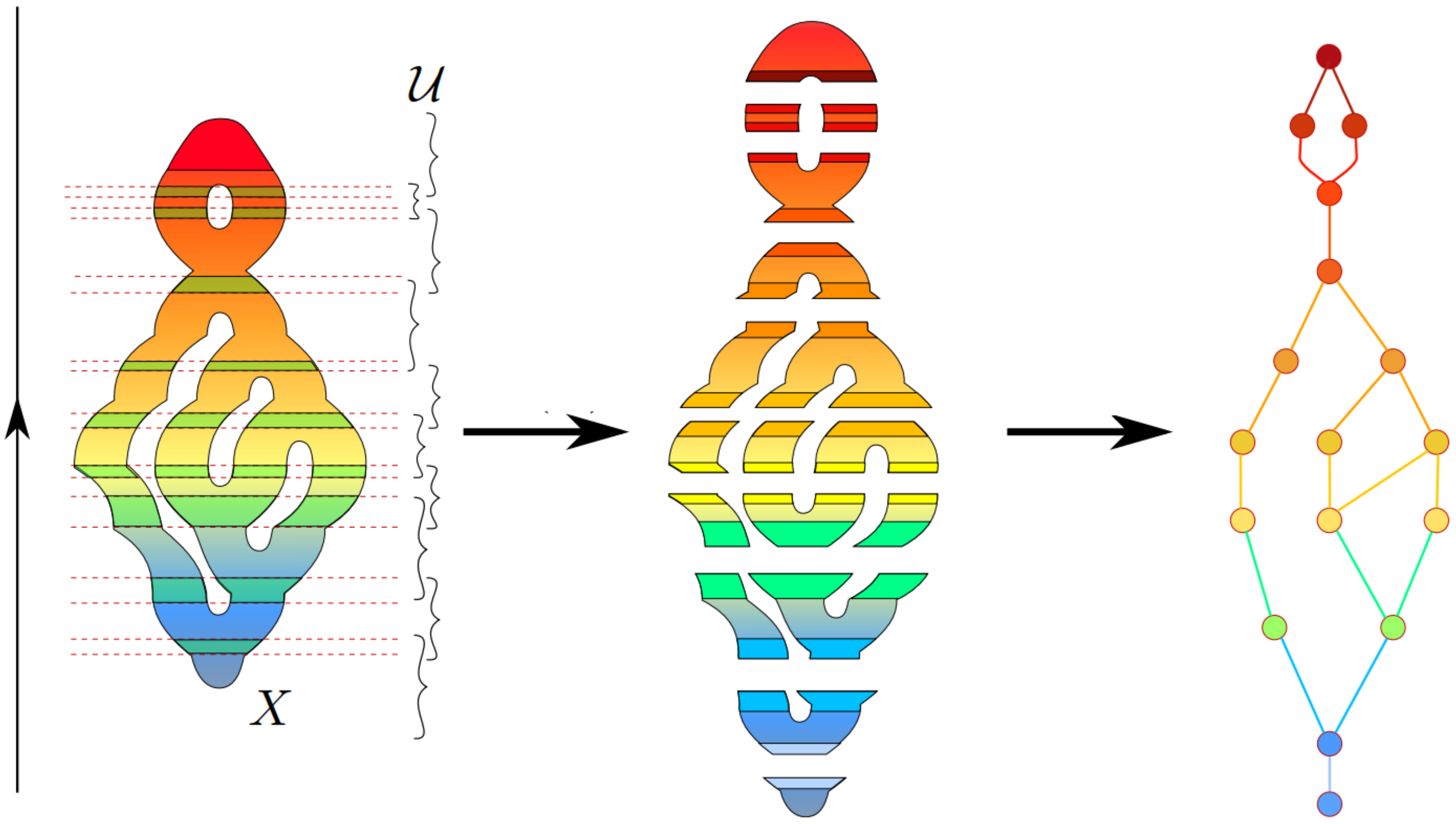
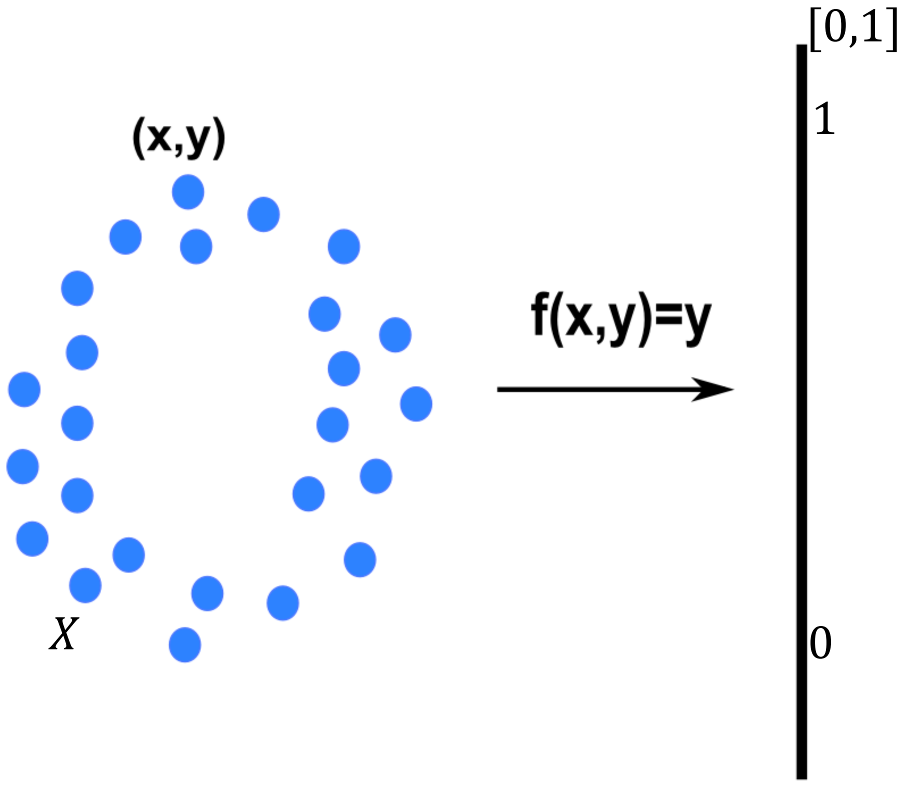


Mapper Example

Suppose that we are given a data set X and a scalar function $f: X \to [a, b]$ defined on every point in X.



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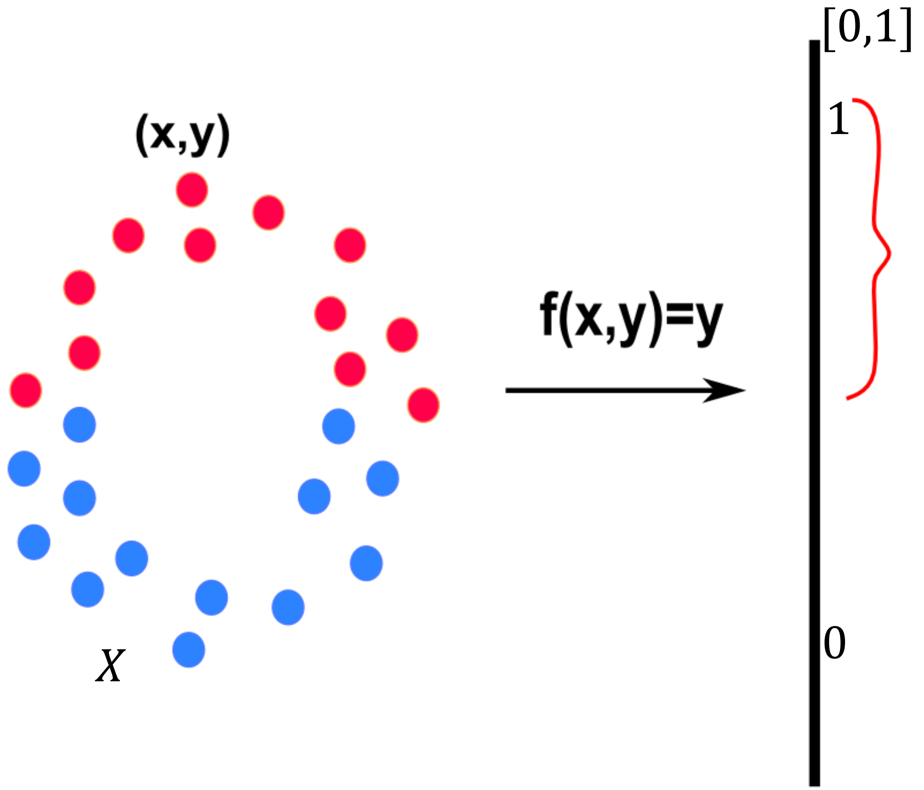

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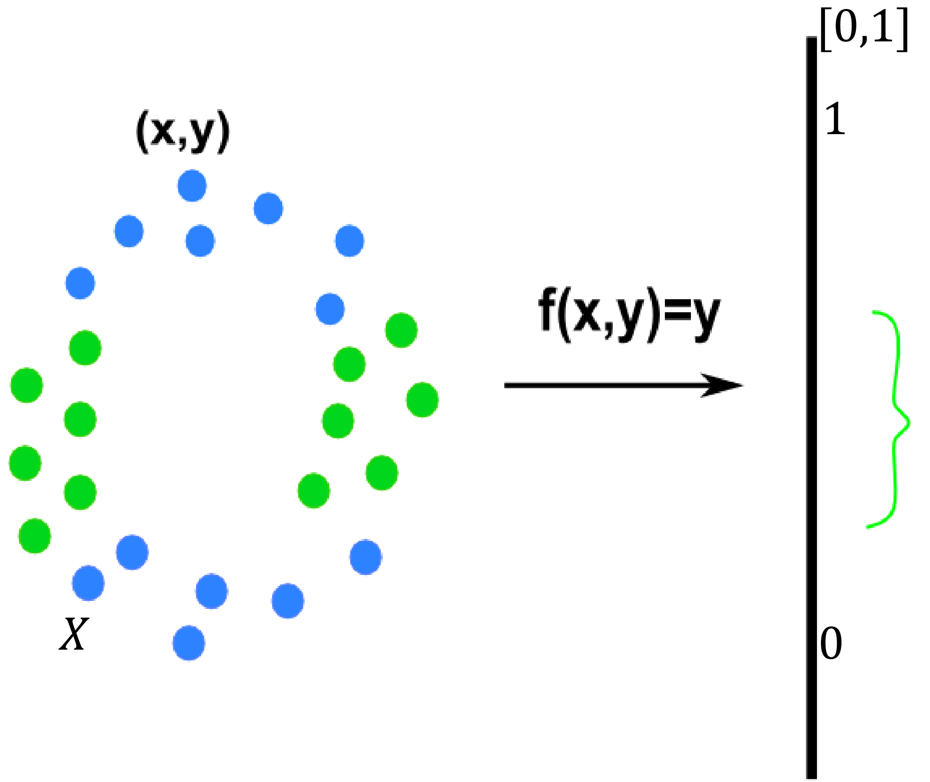

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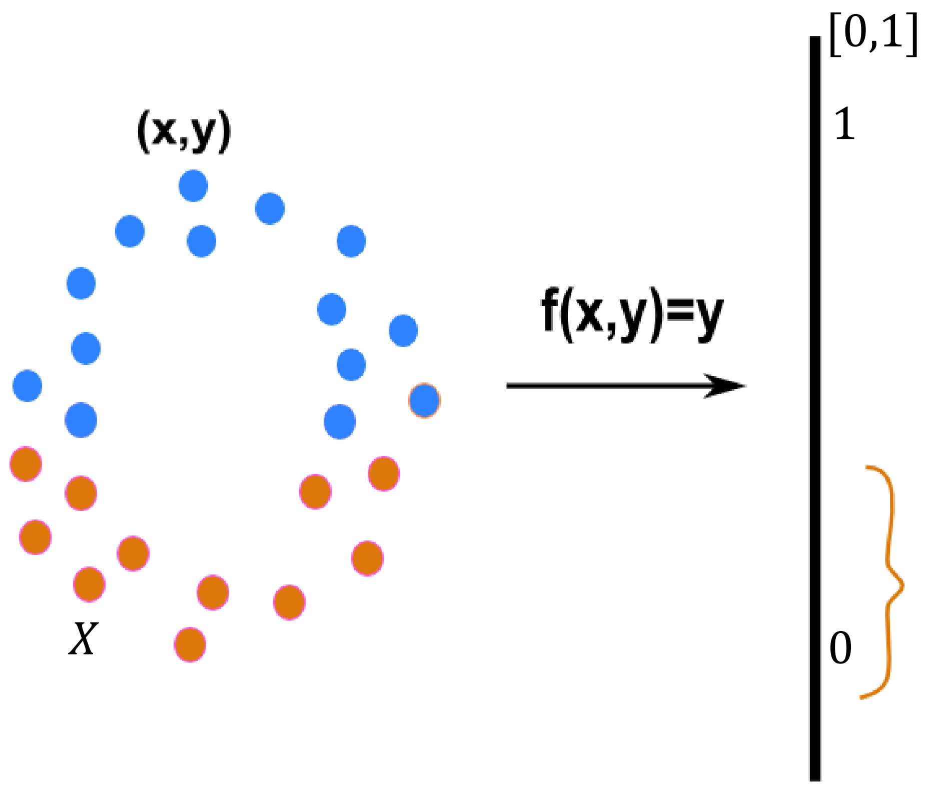

Mapper Example

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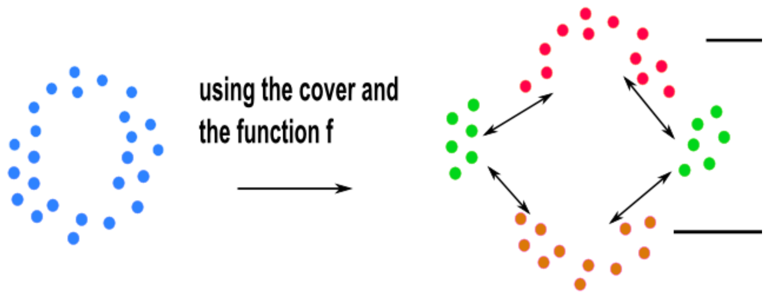

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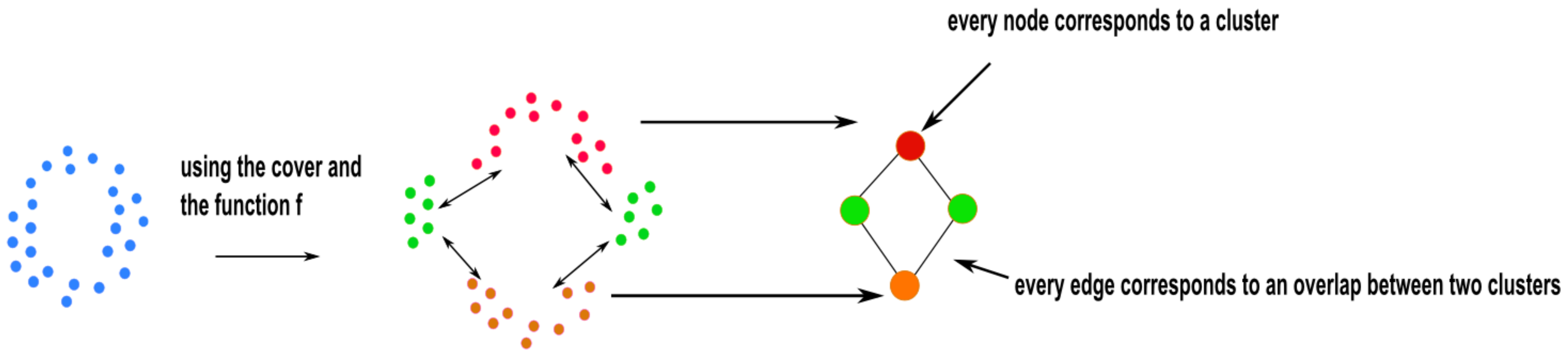

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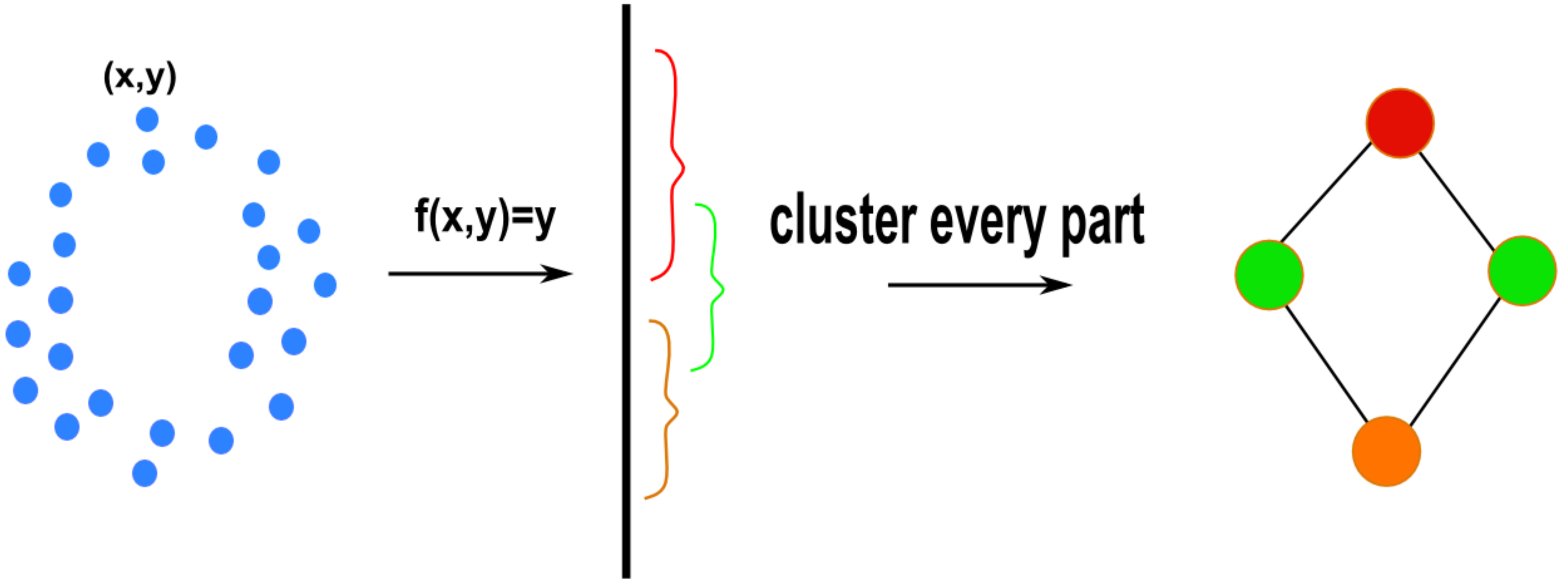
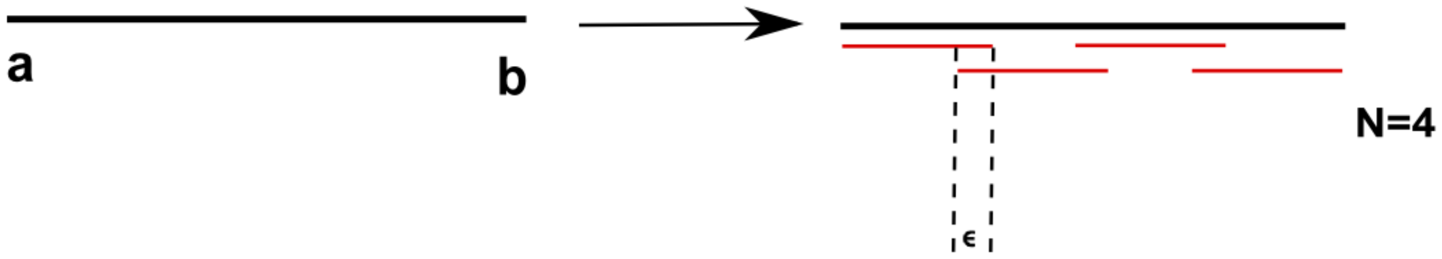


Cover construction algorithm

Given an interval [a,b] we want to divide [a,b] into N segments each two adjacent ones overlap by an amount **ϵ.**



For example if [a,b]=[0,1], N=4 and ϵ=0.1 then :

The cover consists of the following intervals : [0,0.25+0.05],
[0.25-0.05,0.5+0.05],
[0.5-0.05,0.75+0.05],
and [0.75-0.05,1]

The Mapper Algorithm

Suppose that we are given a data set X and a scalar function $f: X \rightarrow [a, b]$ defined on every point in X.

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7-Create an empty graph G.

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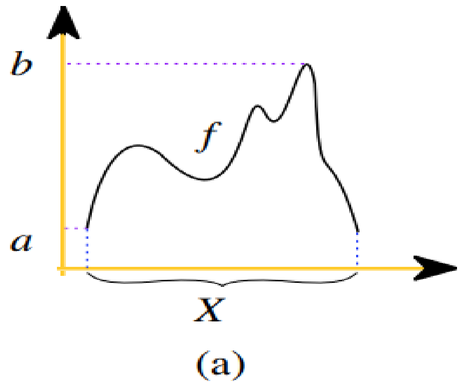
10-return the graph G

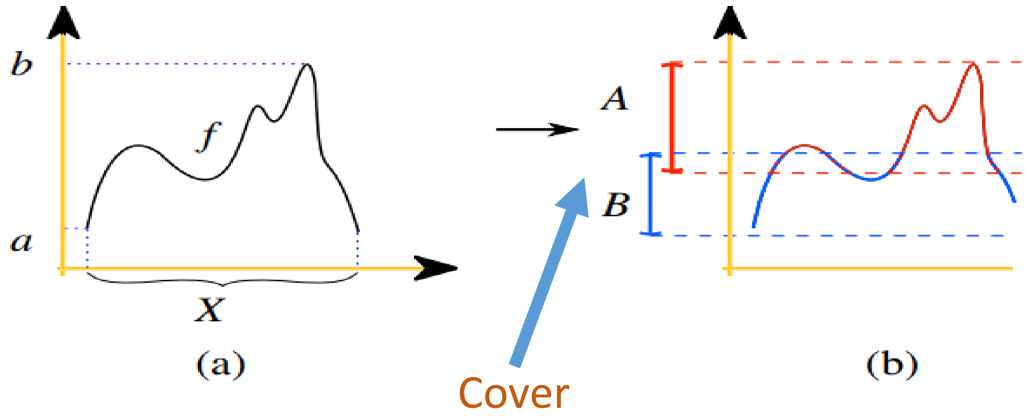
Mapper On Images

The construction of Mapper on a 1d function



(a) A scalar function f : X −→ [a,b].

The construction of Mapper on a 1d function



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(b) The range [a,b] is covered by the two intervals A,B.

I

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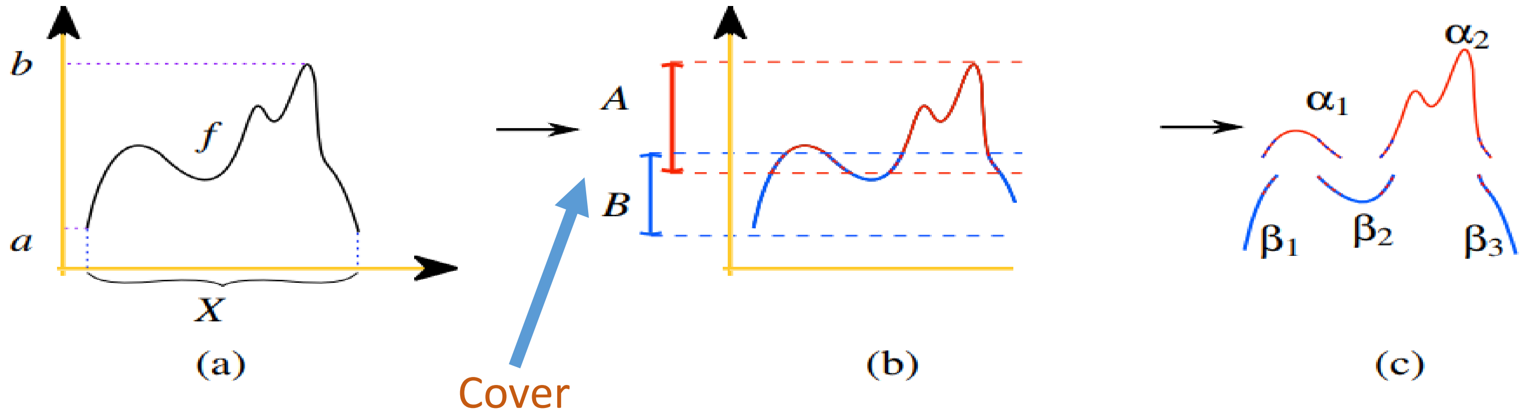

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(c) This gives a decomposition of the domain the domain X. The inverse image of A consists of two connected components α1 and α2, and the inverse image of B consists of three connected components β1, β3 and β3.

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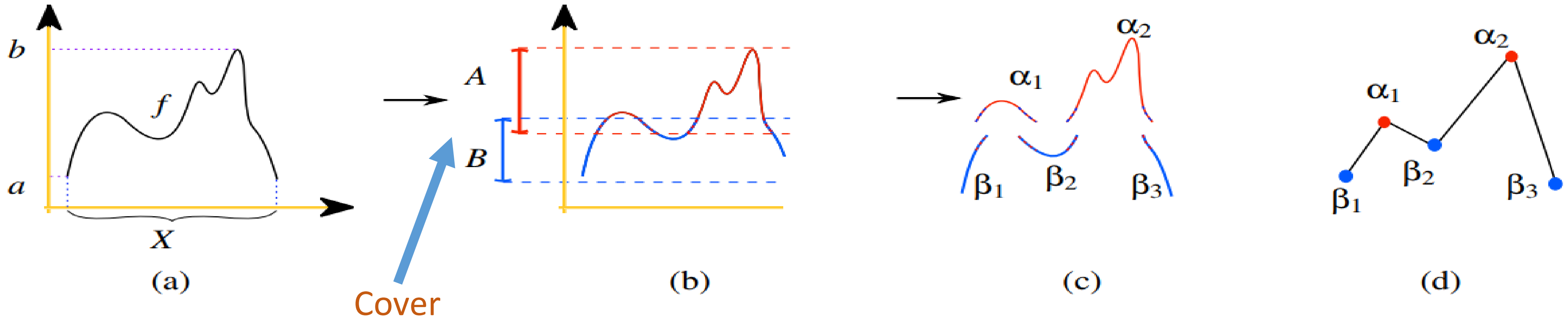


(a) A scalar function f : X –→ [a,b].

(b) The range [a,b] is covered by the two intervals A,B.

(c) This gives a decomposition of the domain the domain X. The inverse image of A consists of two connected components $\alpha 1$ and $\alpha 2$, and the inverse image of B consists of three connected components $\beta 1$, $\beta 3$ and $\beta 3$.

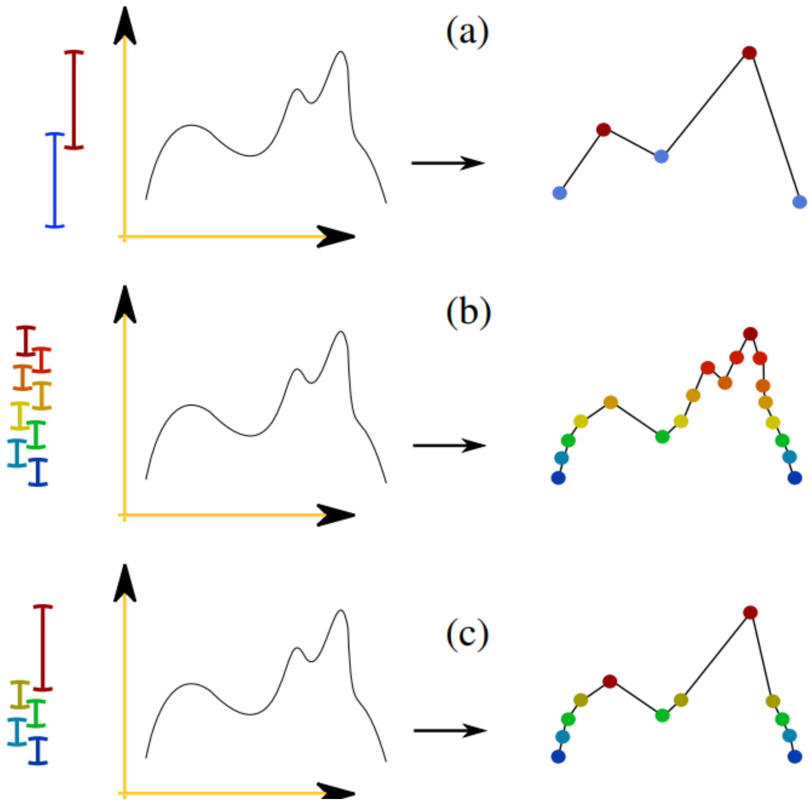
(d) The connected components are represented by the nodes in the Mapper construction.

an edge is inserted whenever two connected components overlap.

Mapper resolution

The construction of mapper depends on the cover chosen for the range [a,b] of the scalar function.

The figure shows three different covers for the range [a,b] and each one gives rise to a different resolution of Mapper.



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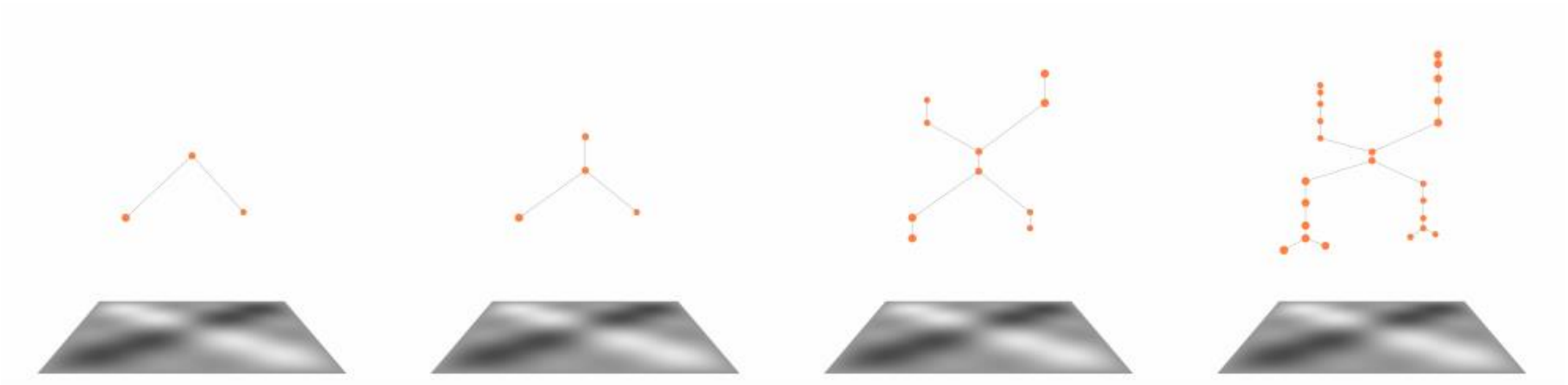
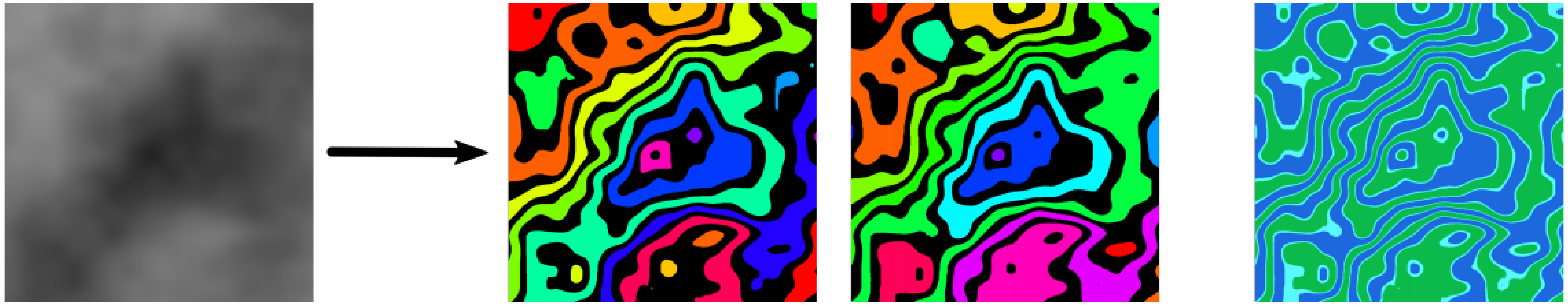


Multi-resolution of Mapper using different cover resolutions. The graphs are constructed from left to right by using 2,4,8,16 slices of the range cover.

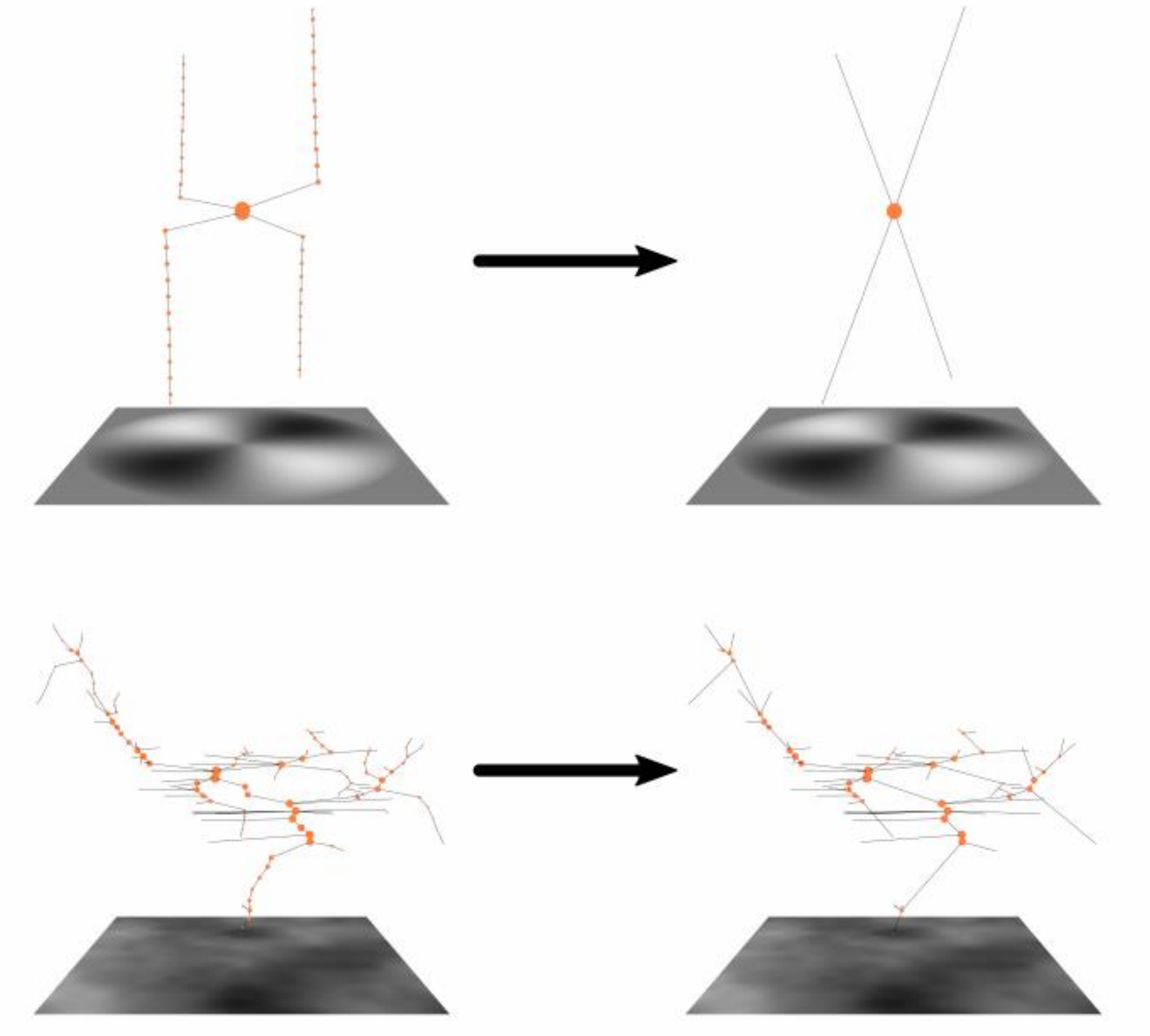
Mapper on images



Input Even Odd Overlap

A node in Mapper is a connected component of $f^{-1}((c, d))$, where (c, d) is an open interval in the cover U of the range of f.

Mapper on images



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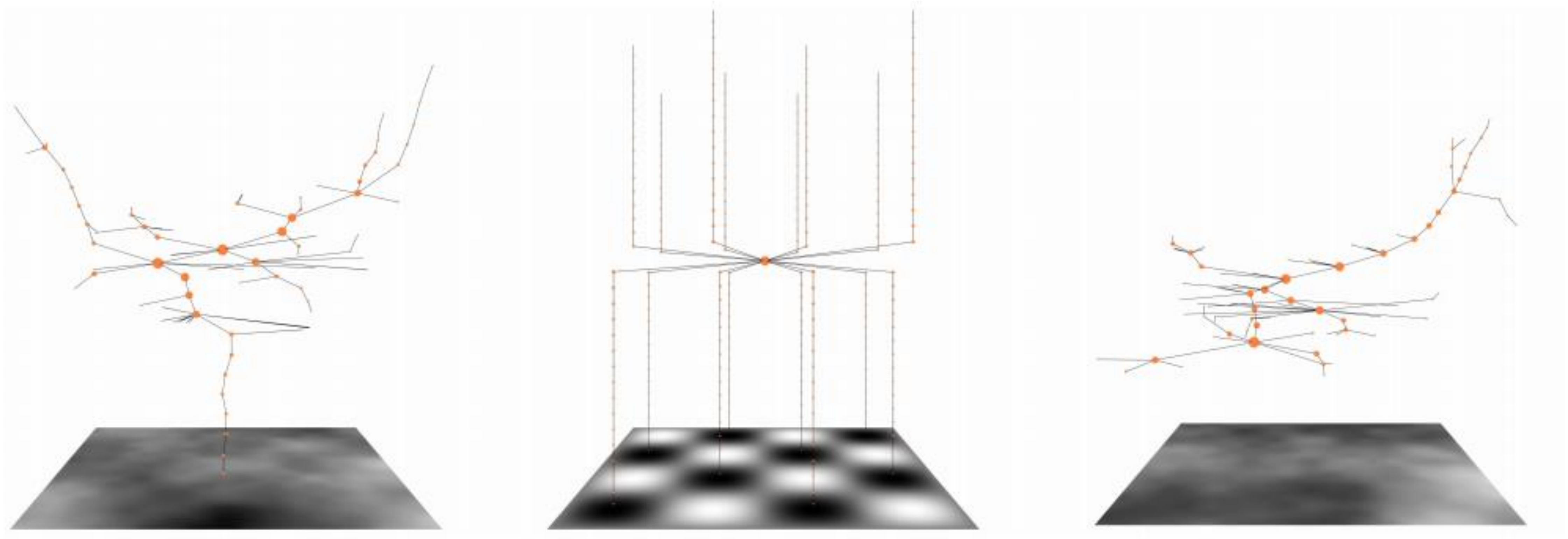
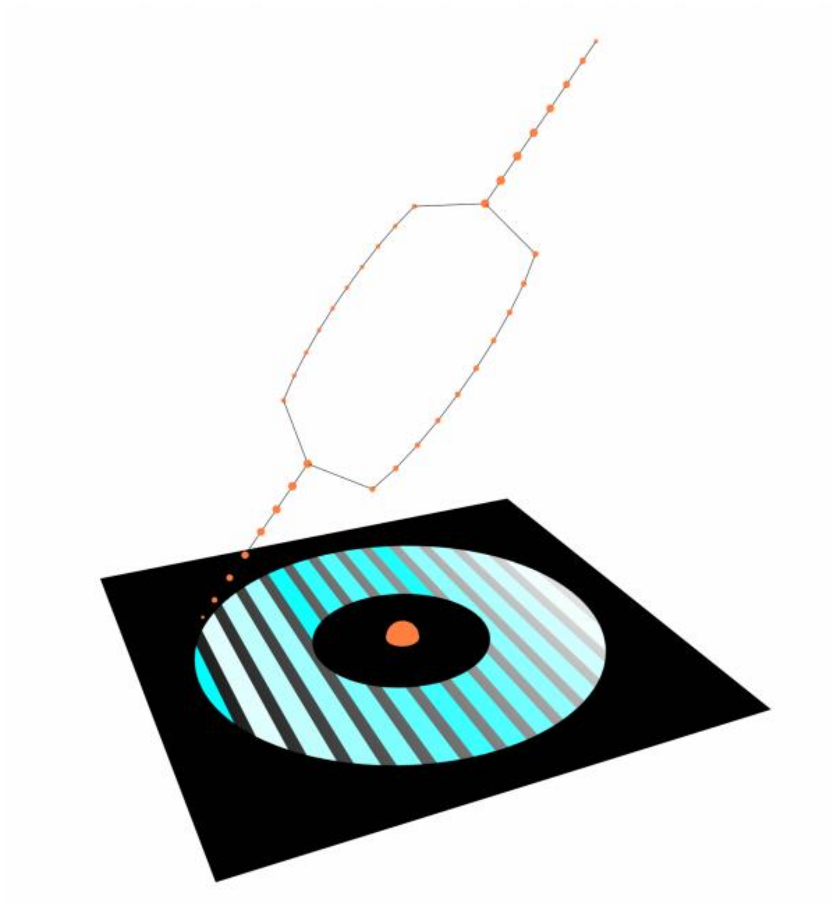
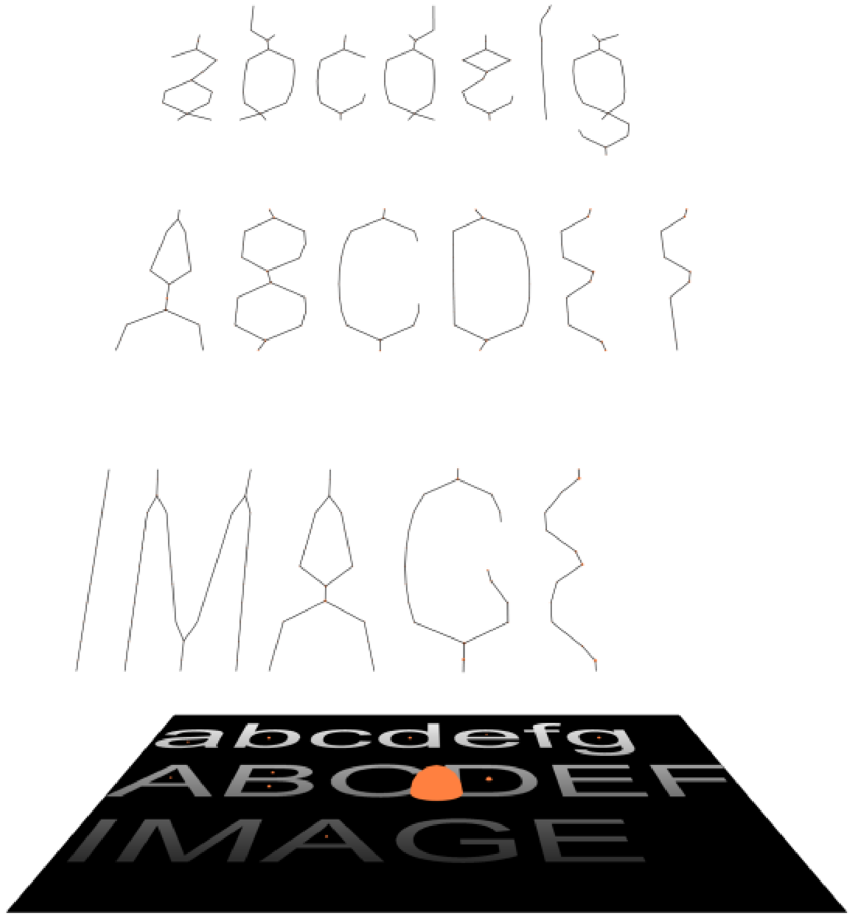


Shape of an image





Mapper on an image consists of letters. The Mapper construction on this image gives a collection of disjoint graphs that have the same shape of as the letters written in the image.