Topological Algorithms-I

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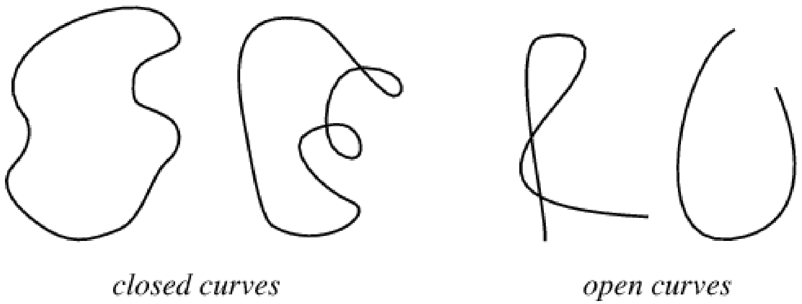

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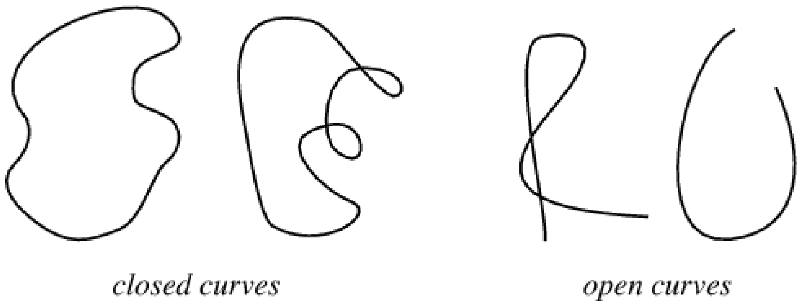

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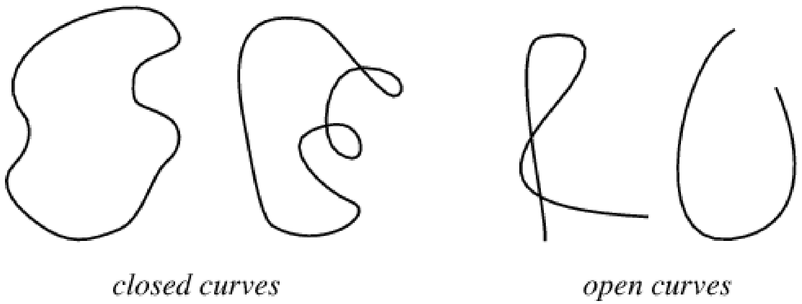

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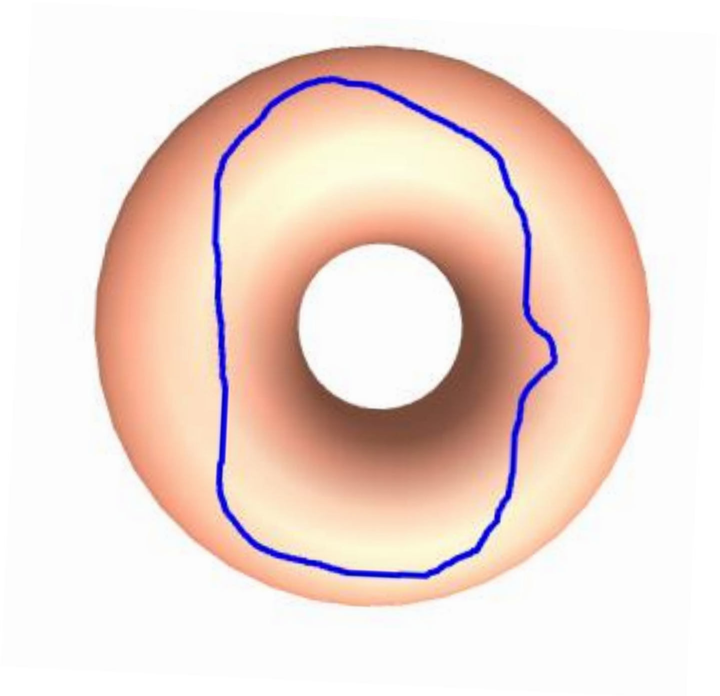

Curve homotopy

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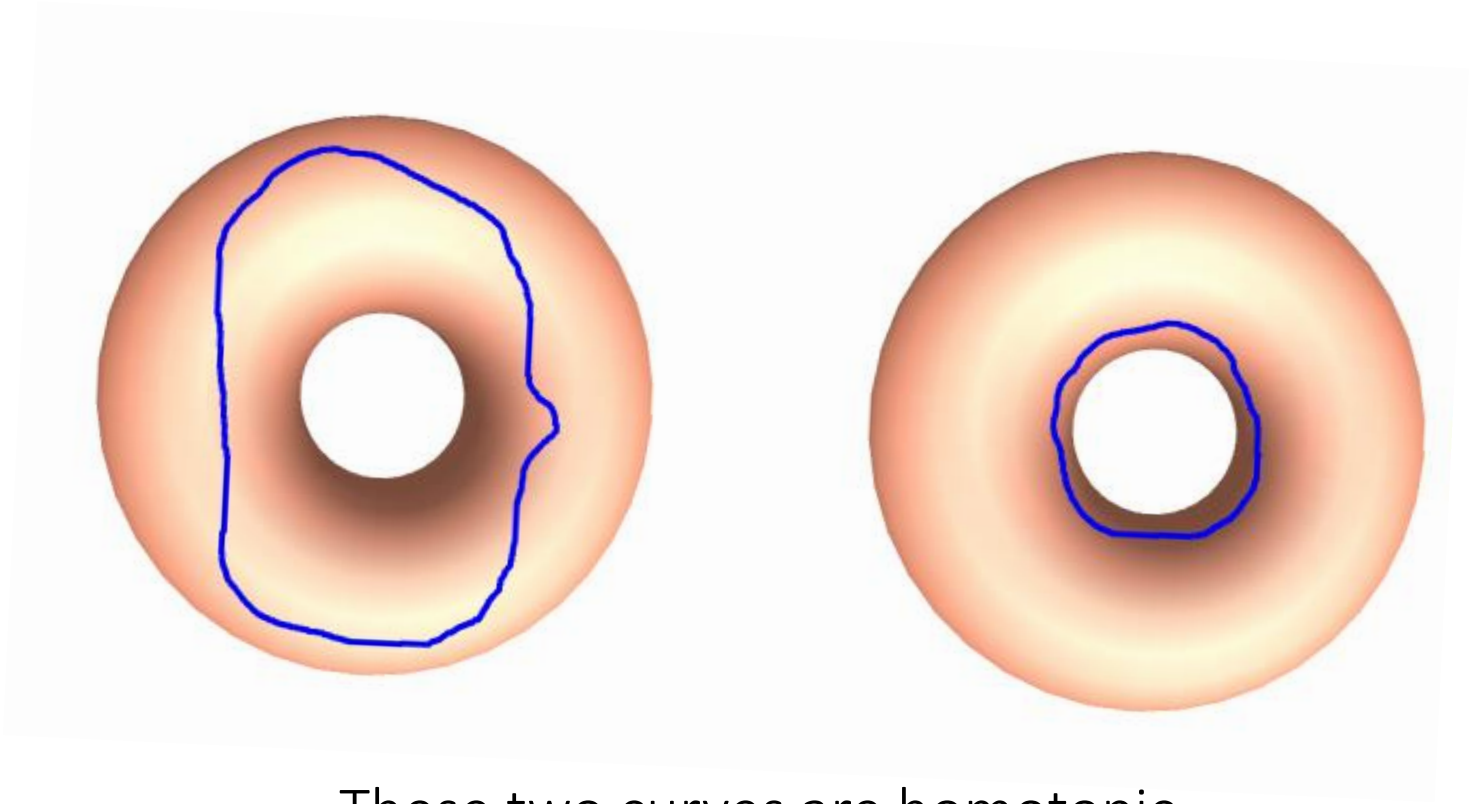

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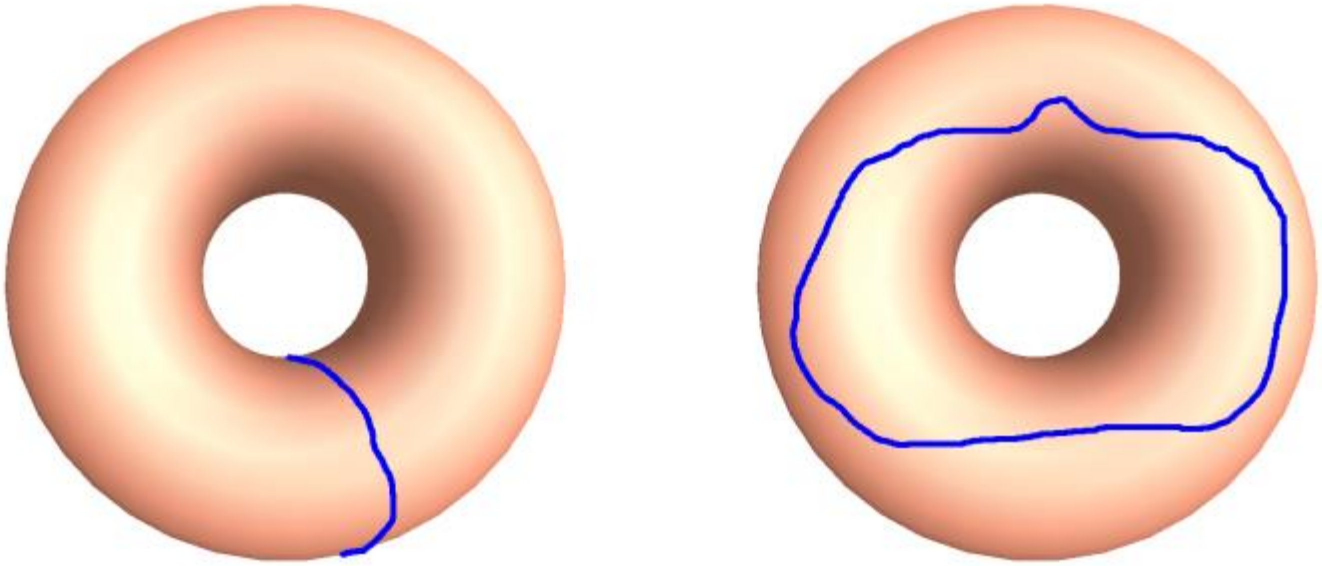

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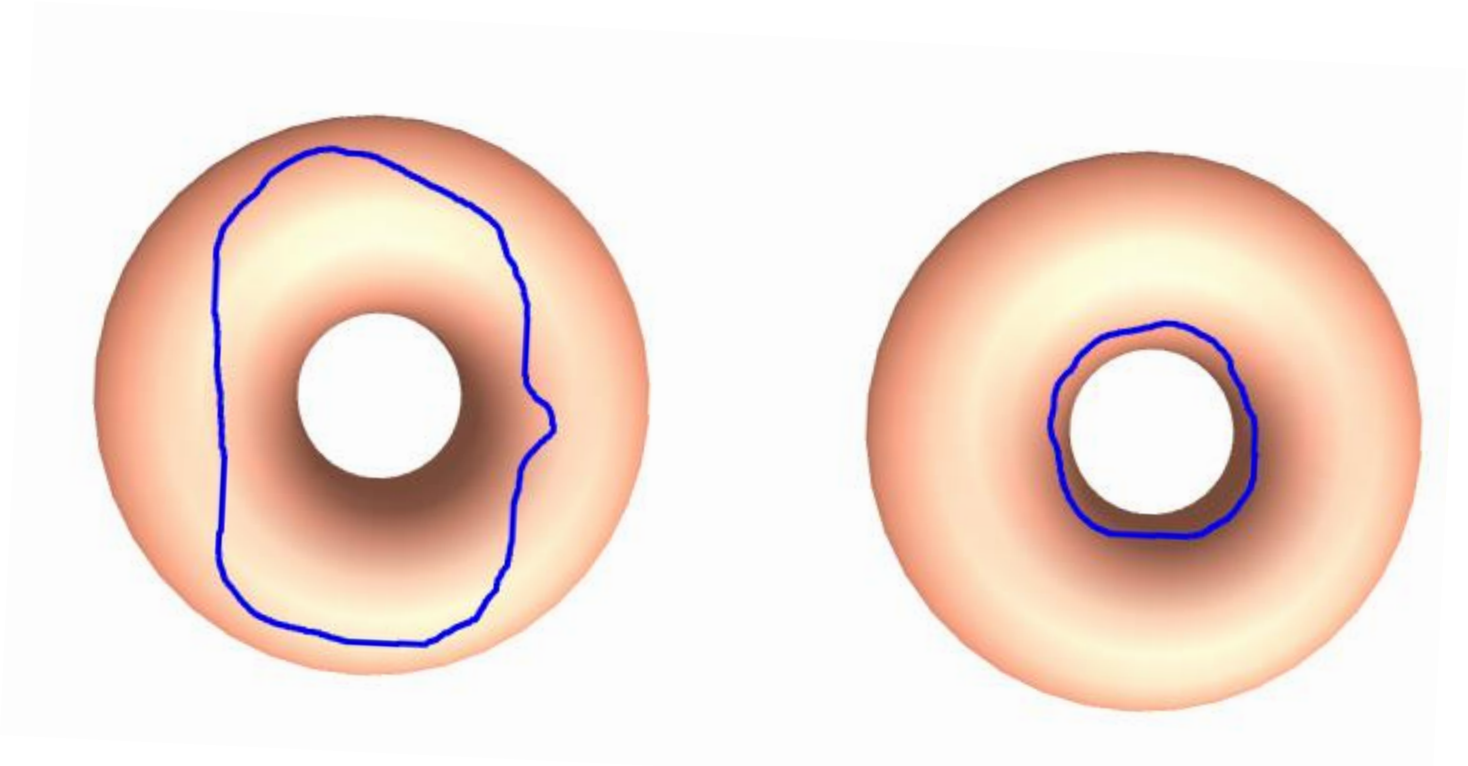

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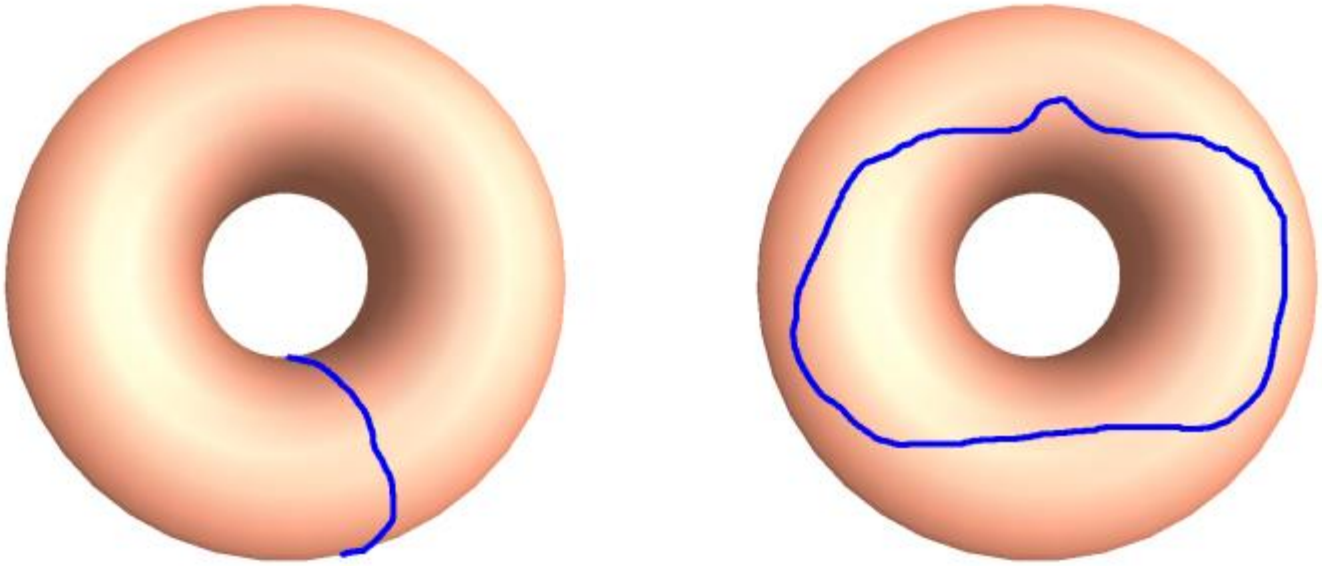

These two curves are in two different homotopy classes

Problems

1-Given two loops on a triangulated surface, determine if they are homotopic.

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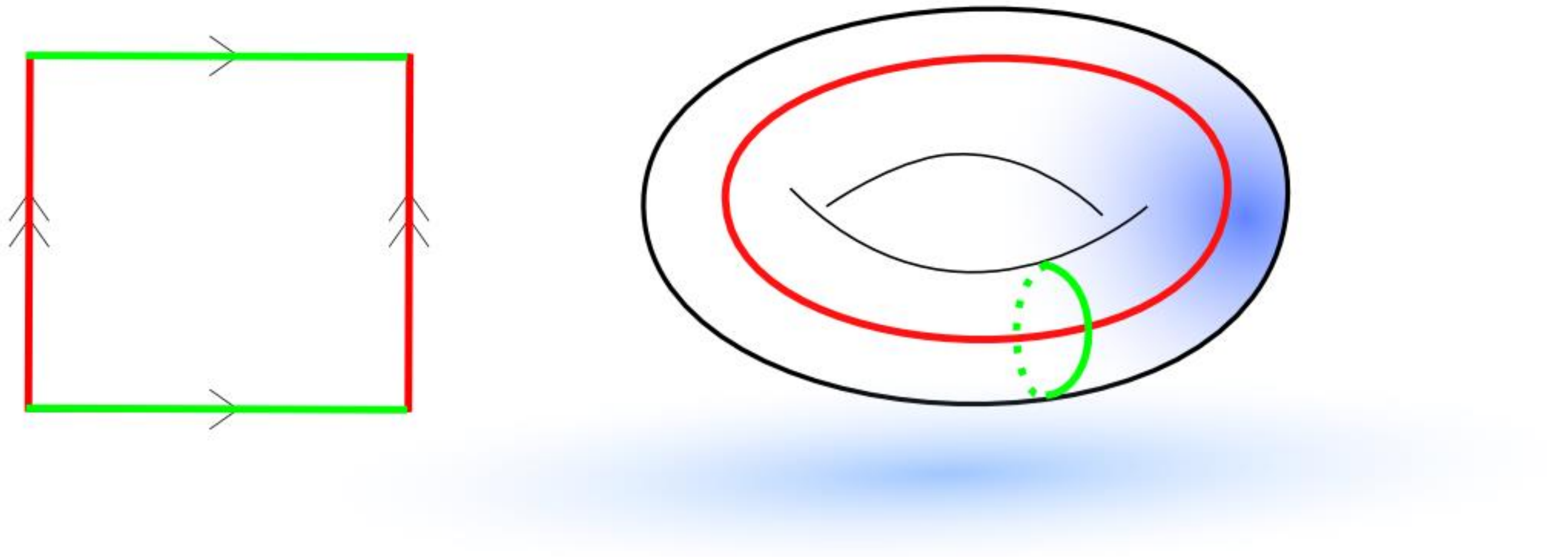
2-Given a loop *a* on surface. Find a loop *b* in the same homotopy class of a with shortest length

Case study



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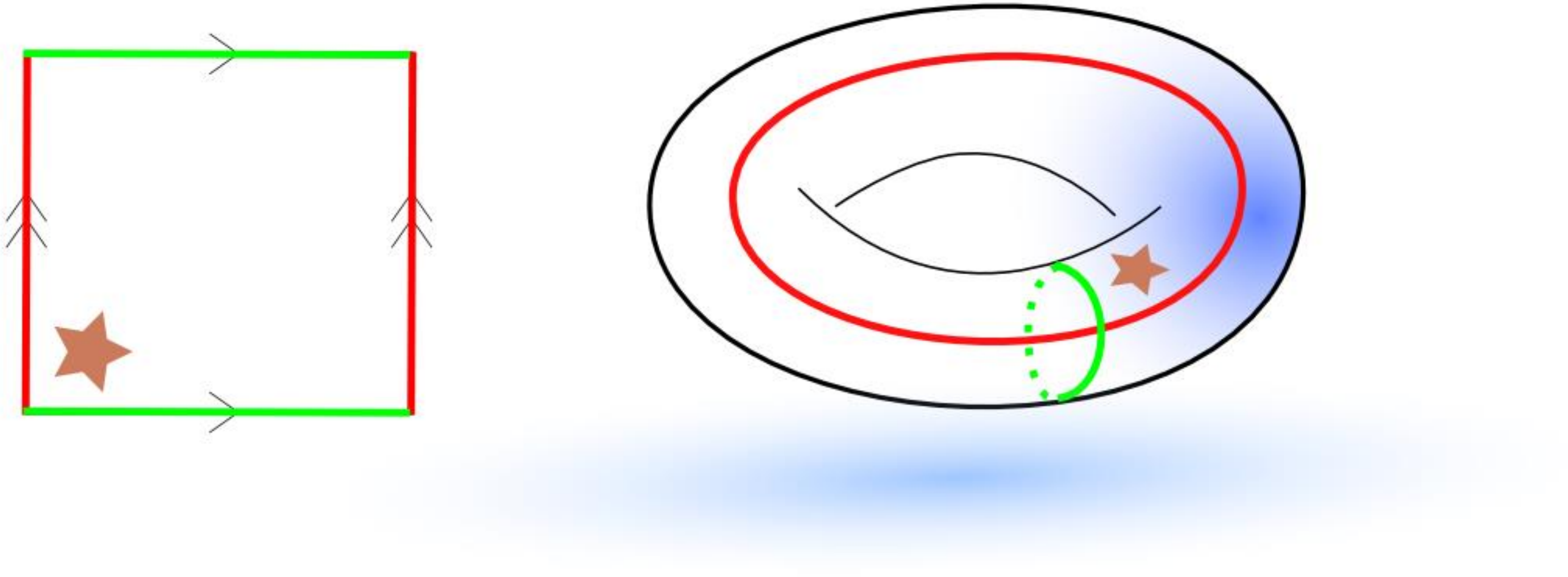

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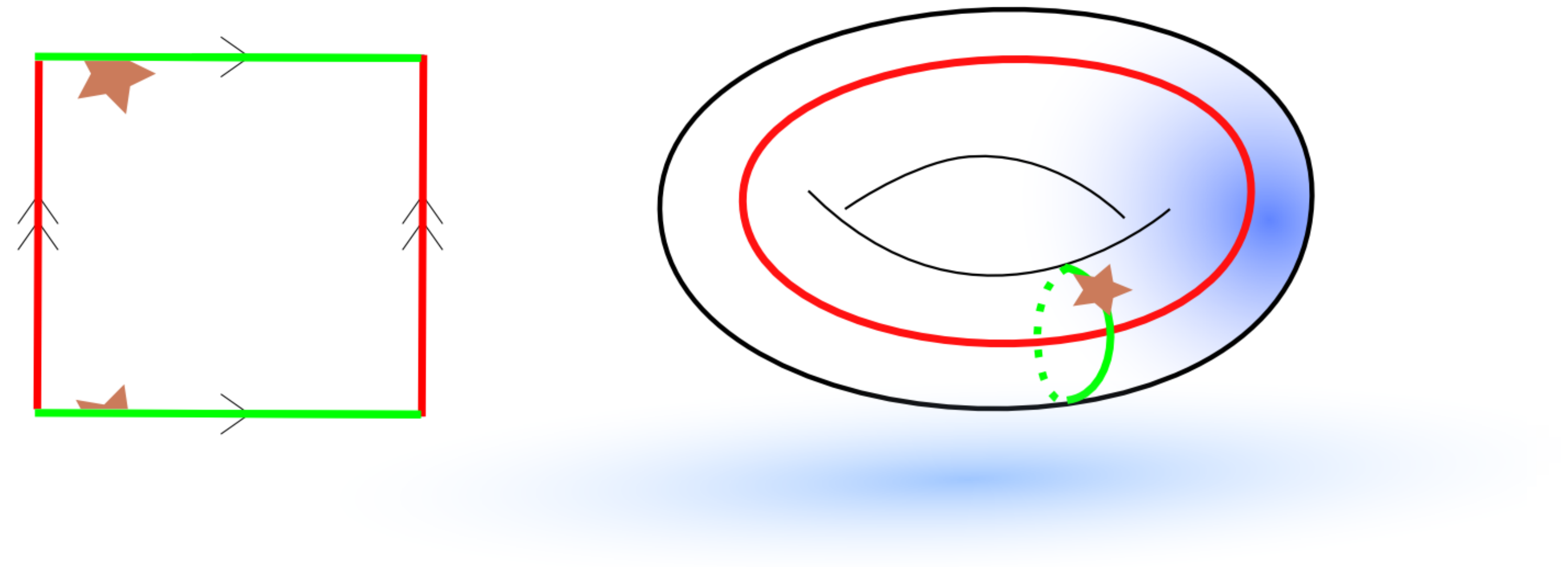

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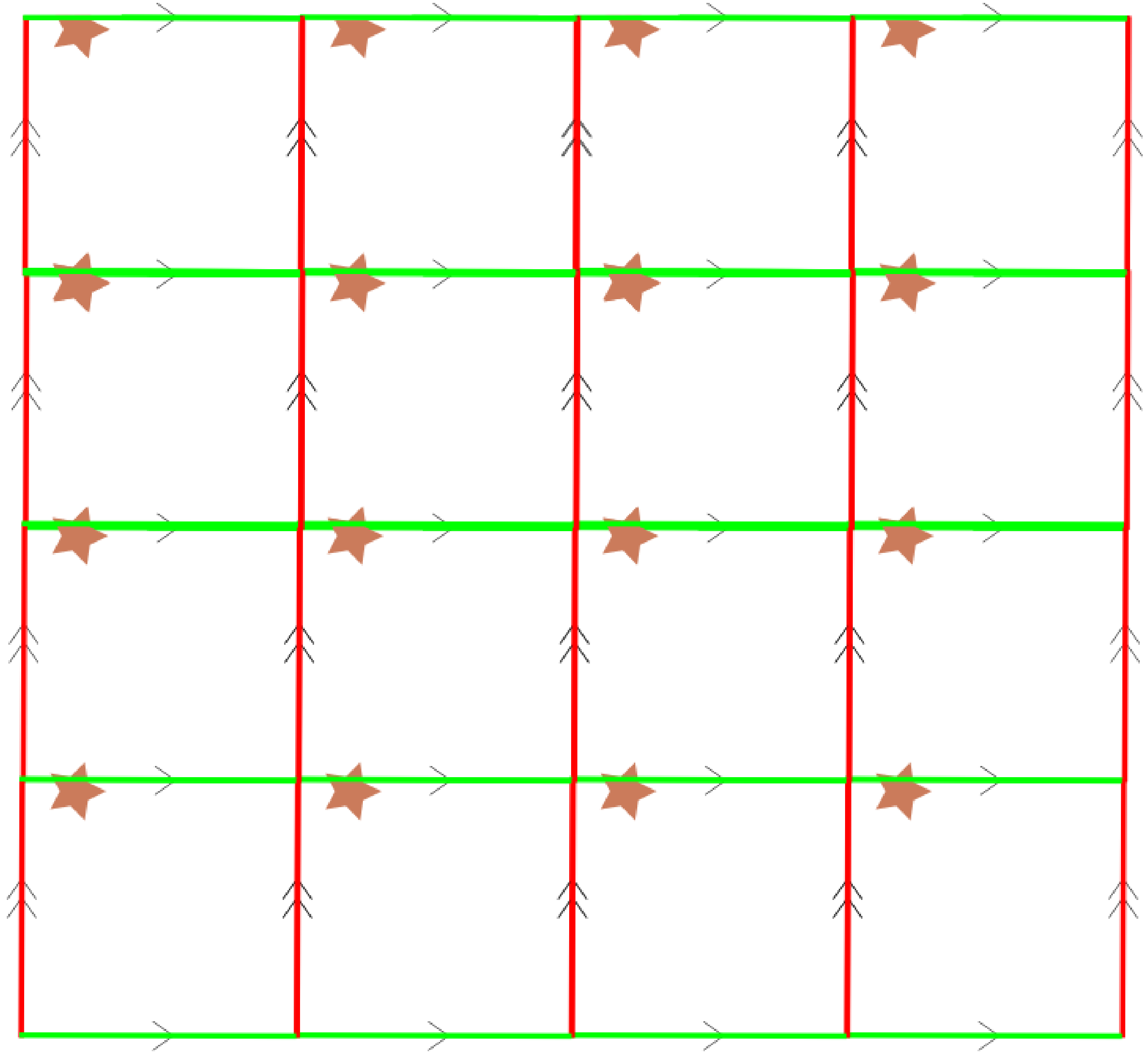

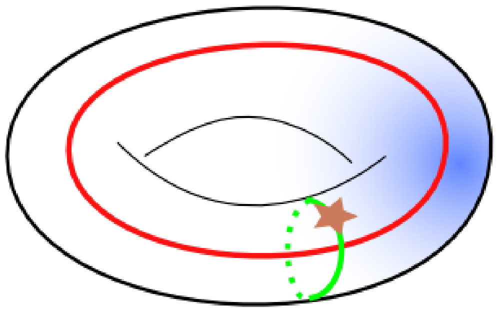

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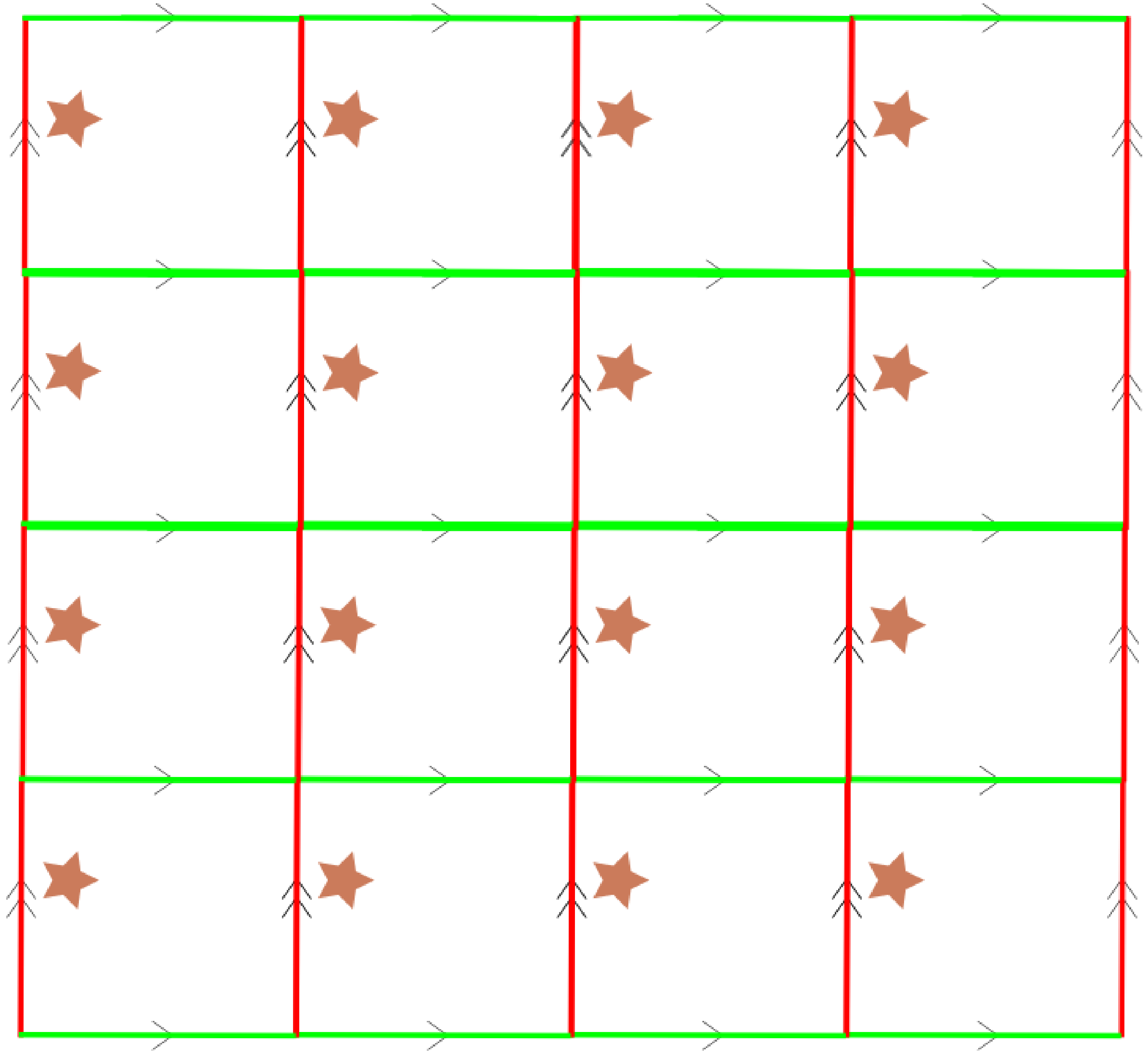

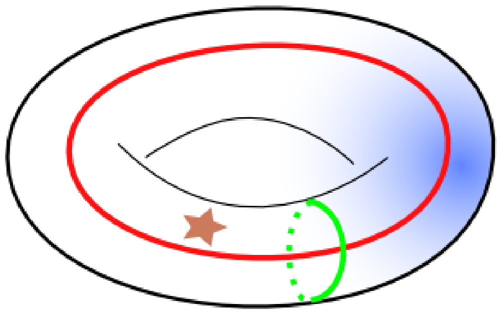

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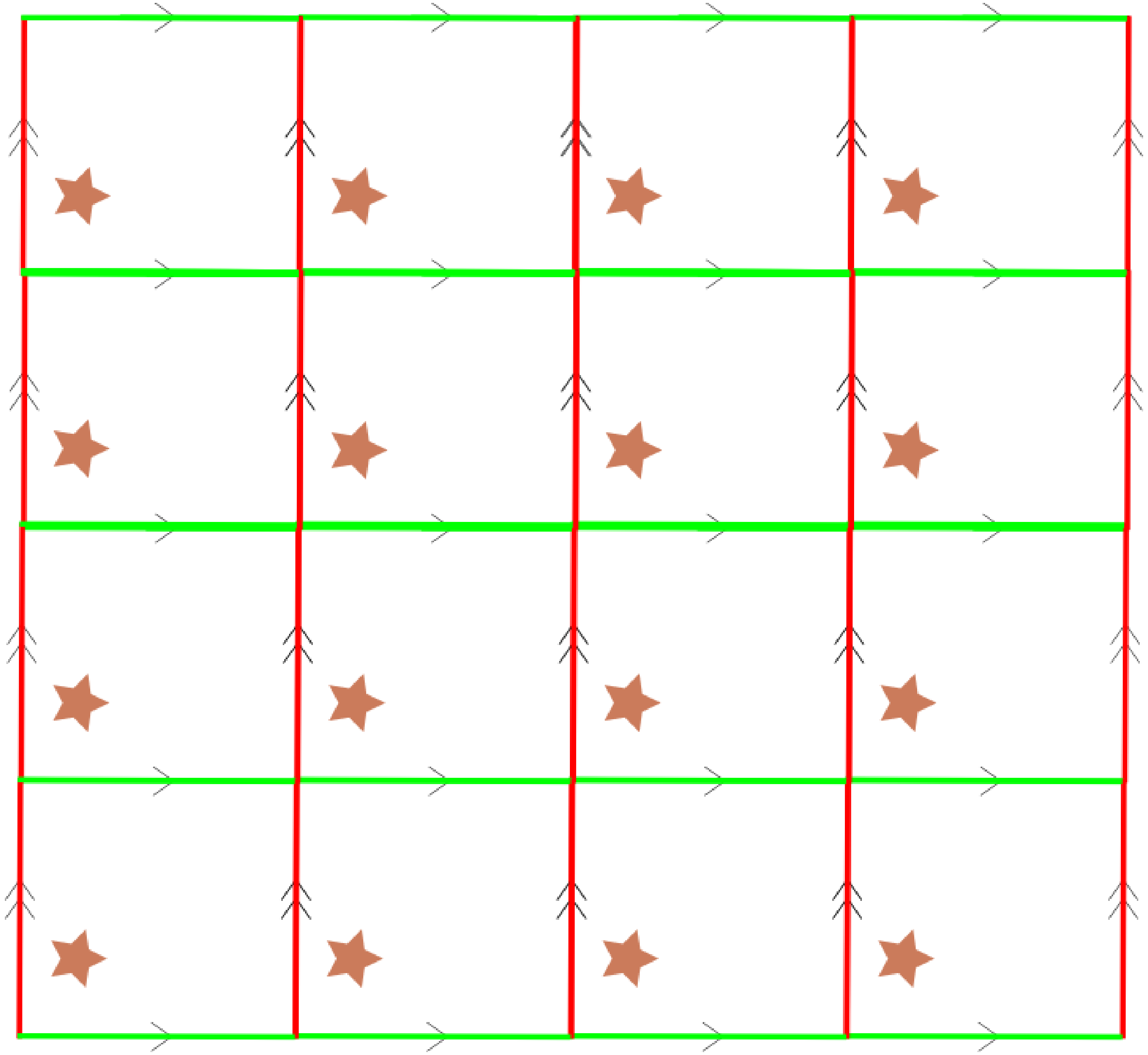

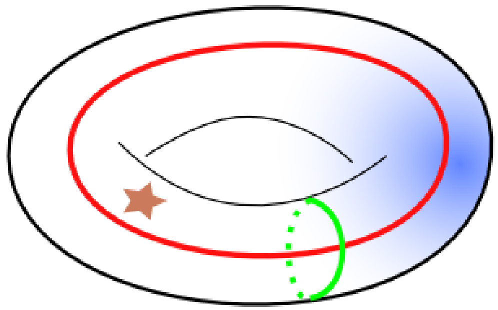

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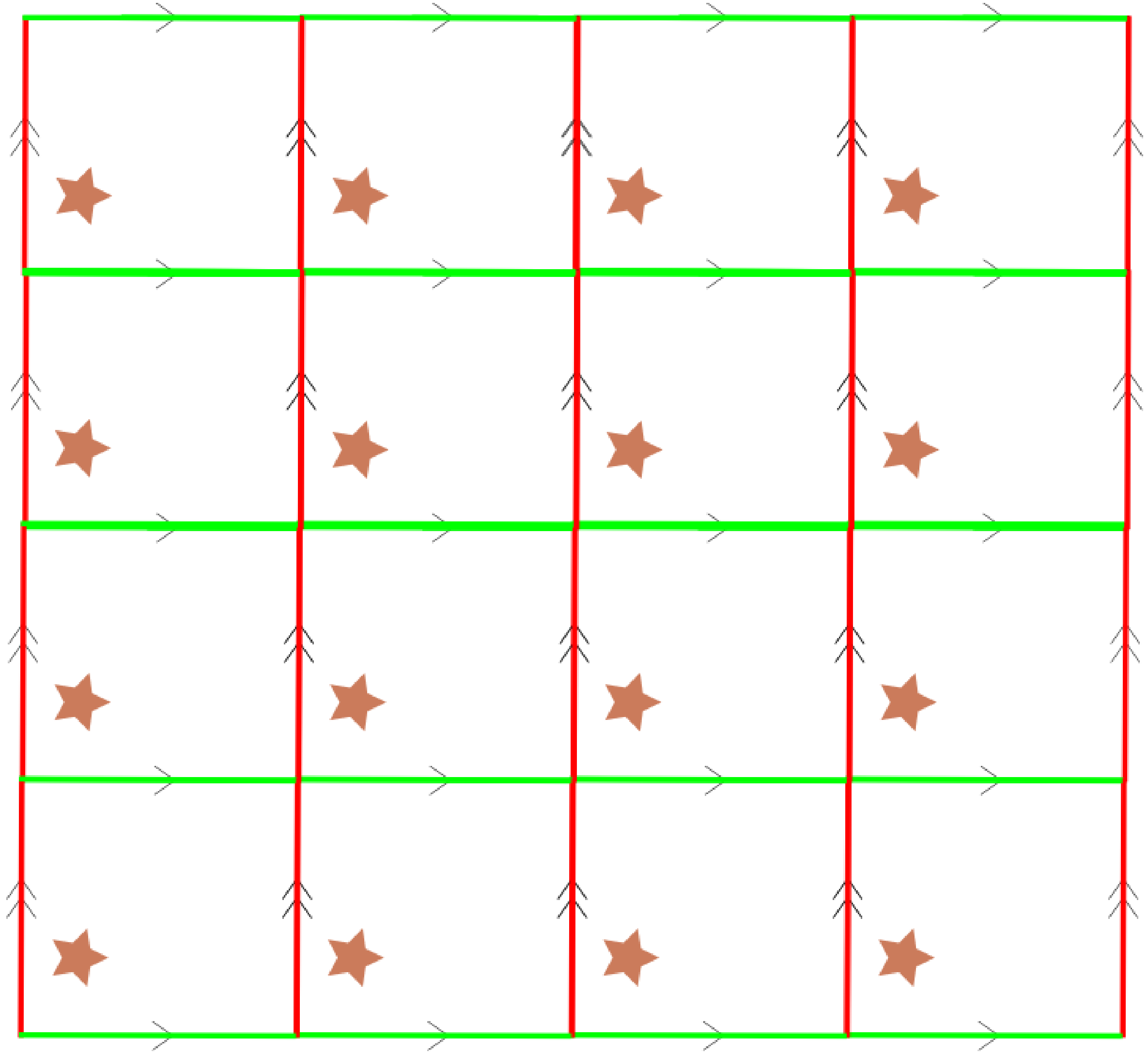

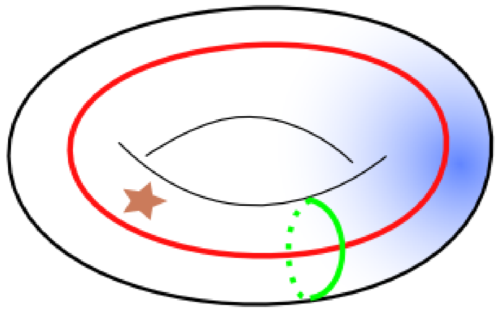

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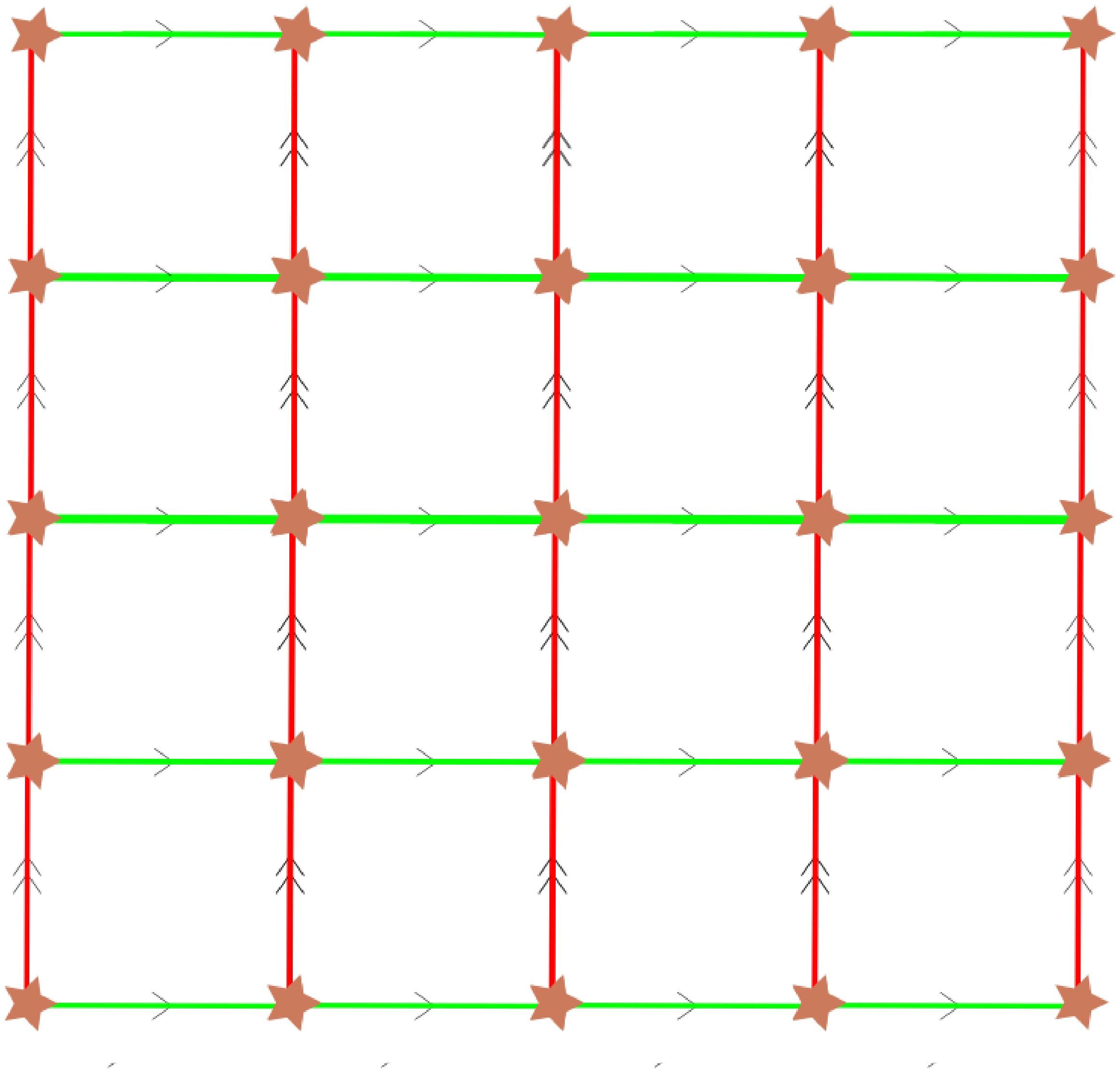

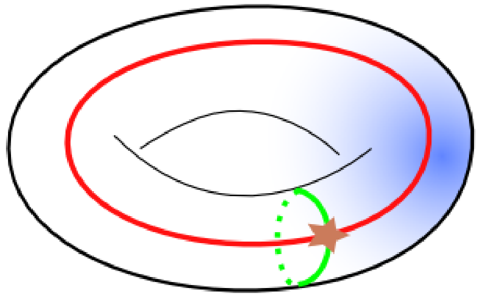

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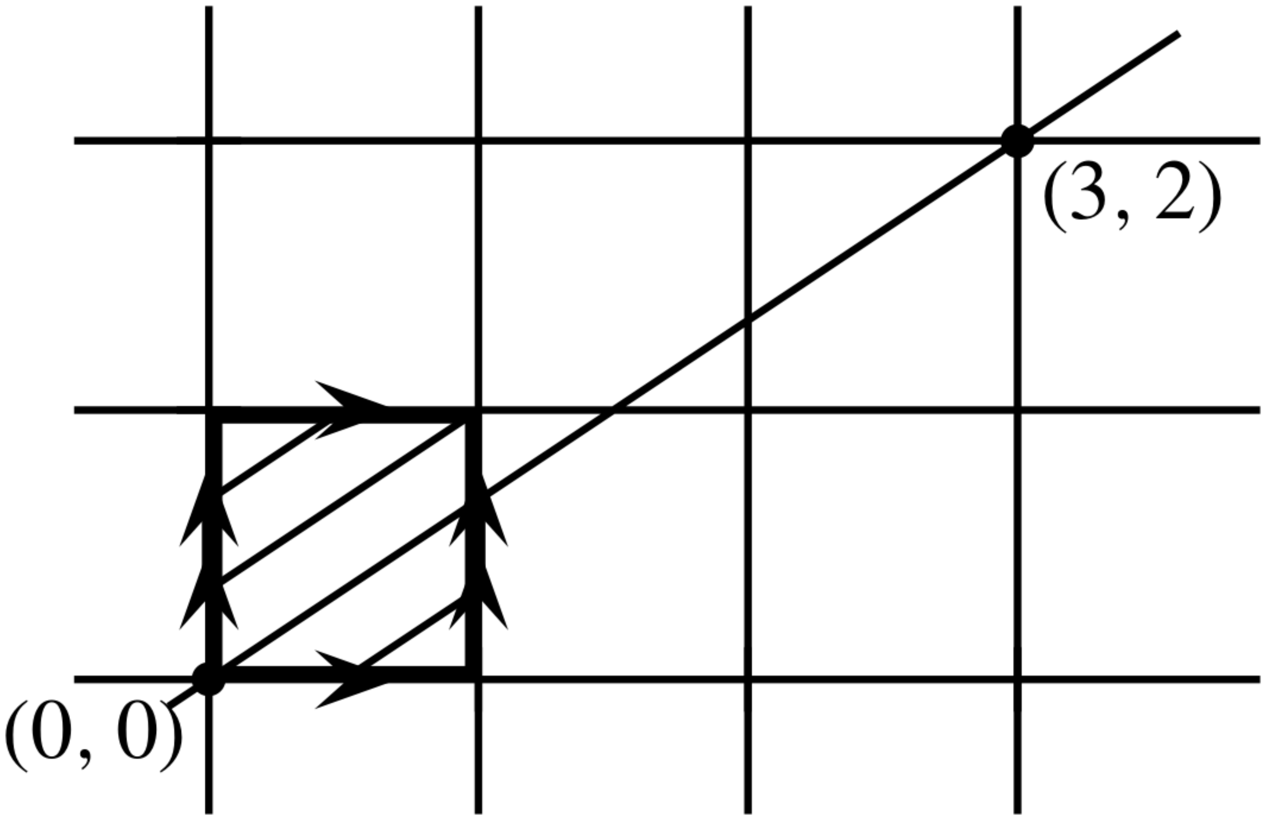

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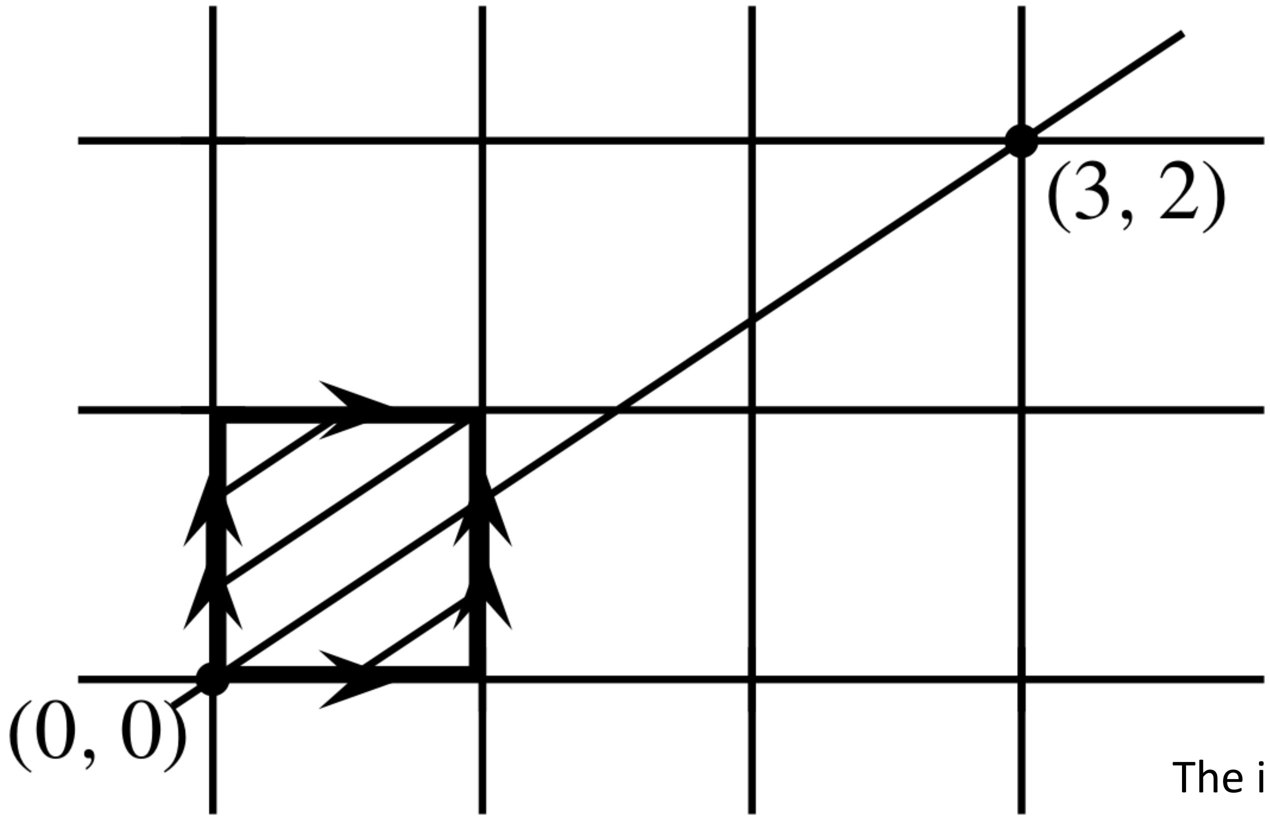

The image of line *L* that passes through the origin is a closed curve on the torus if and only if the line *L* goes through another lattice point, say (m,n) where m and n are integers .

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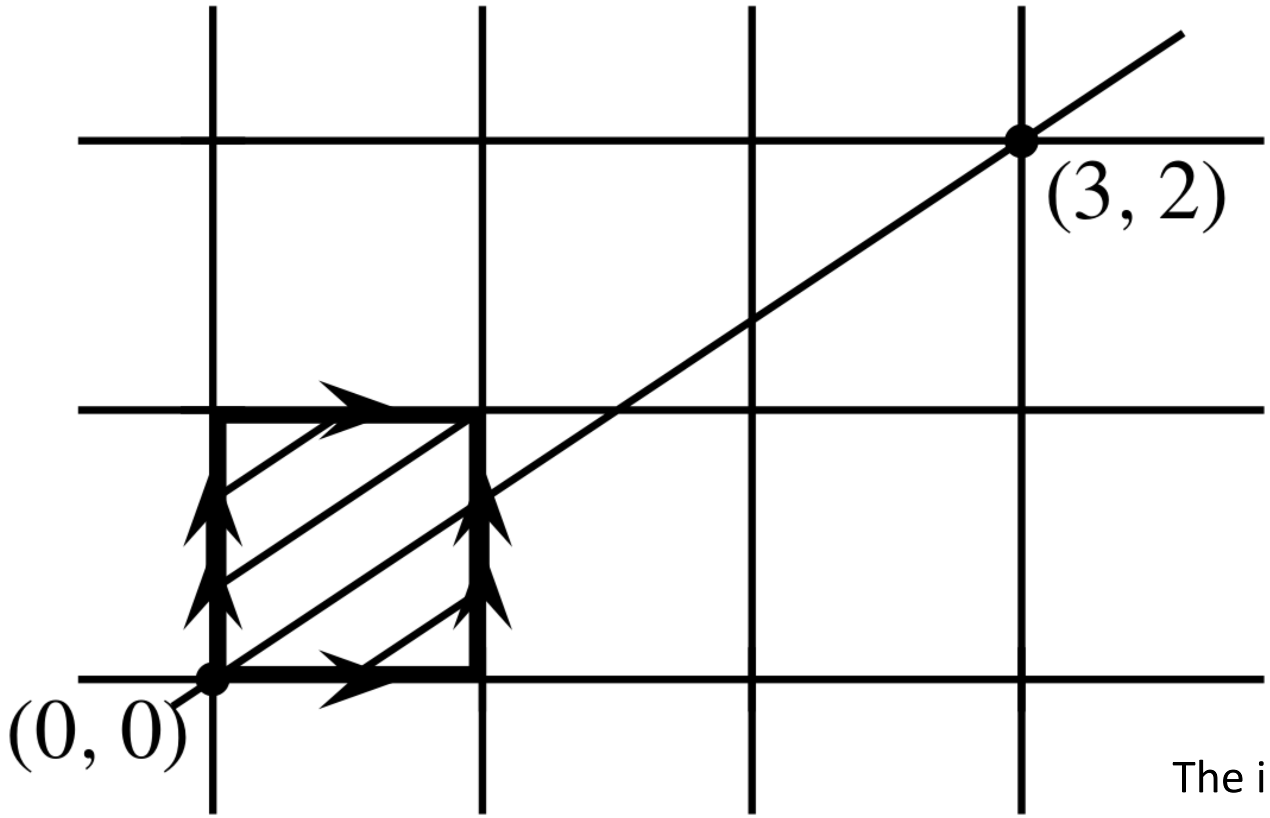

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This is the case if and only if the slope of L is n/m, a rational number.

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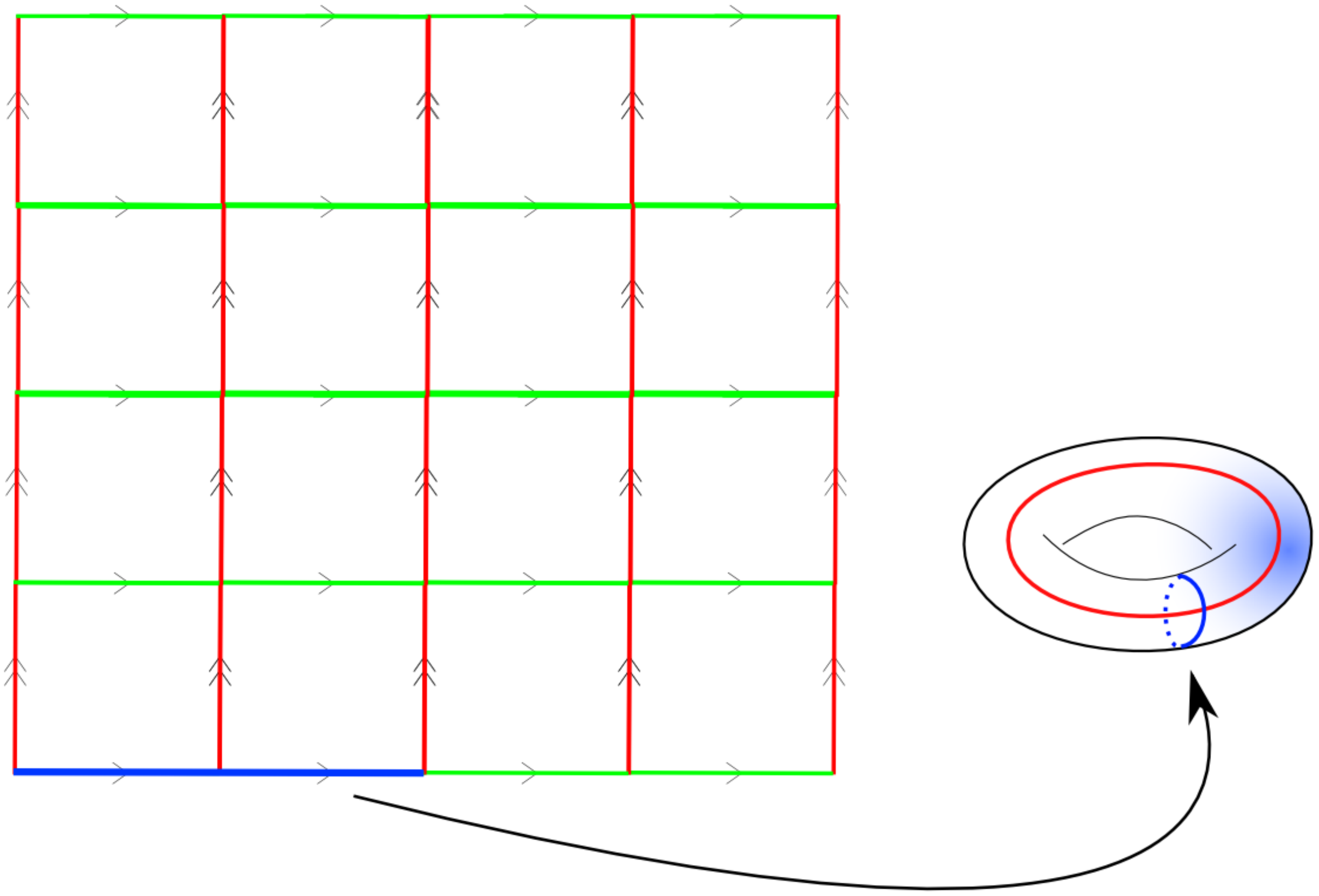

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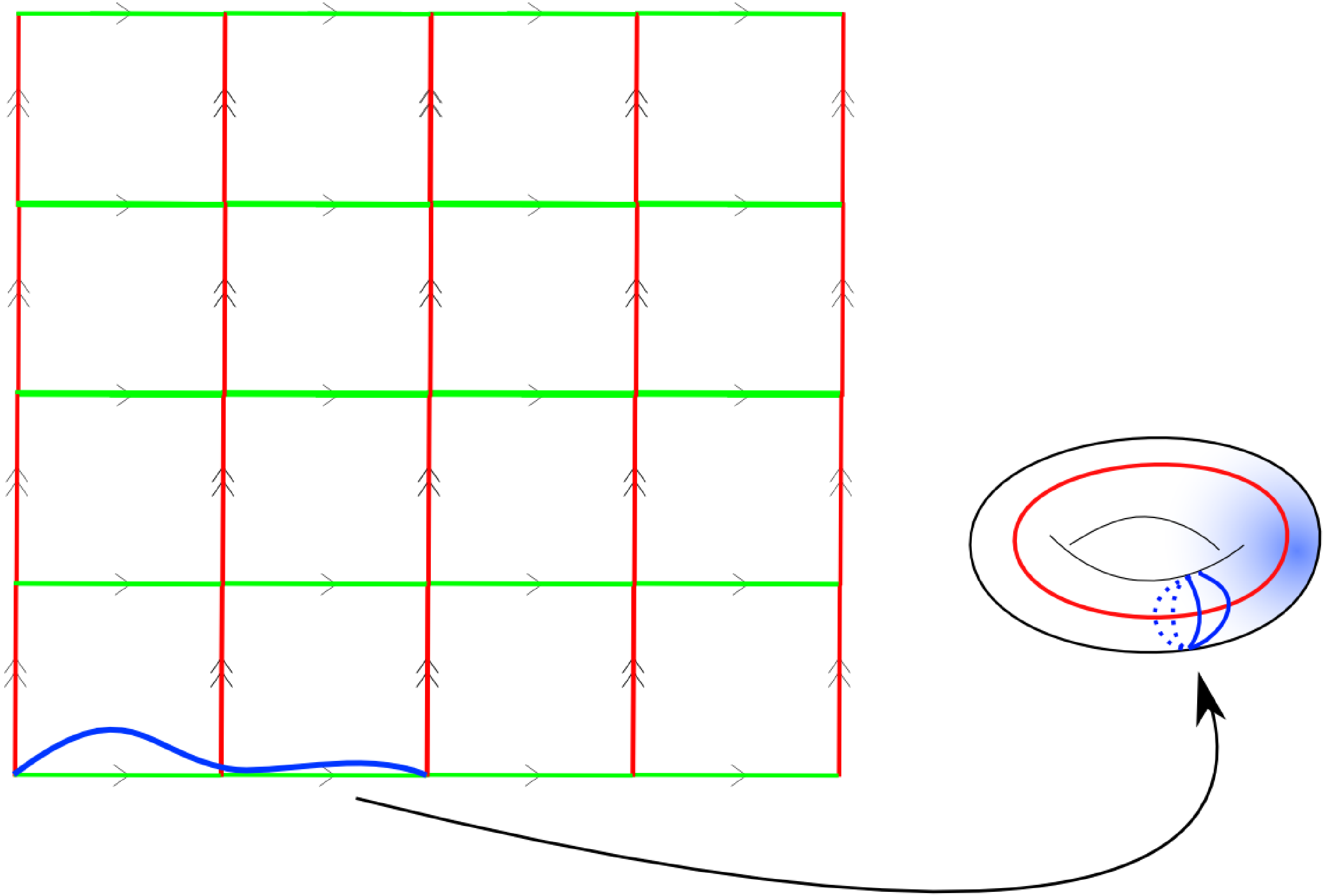

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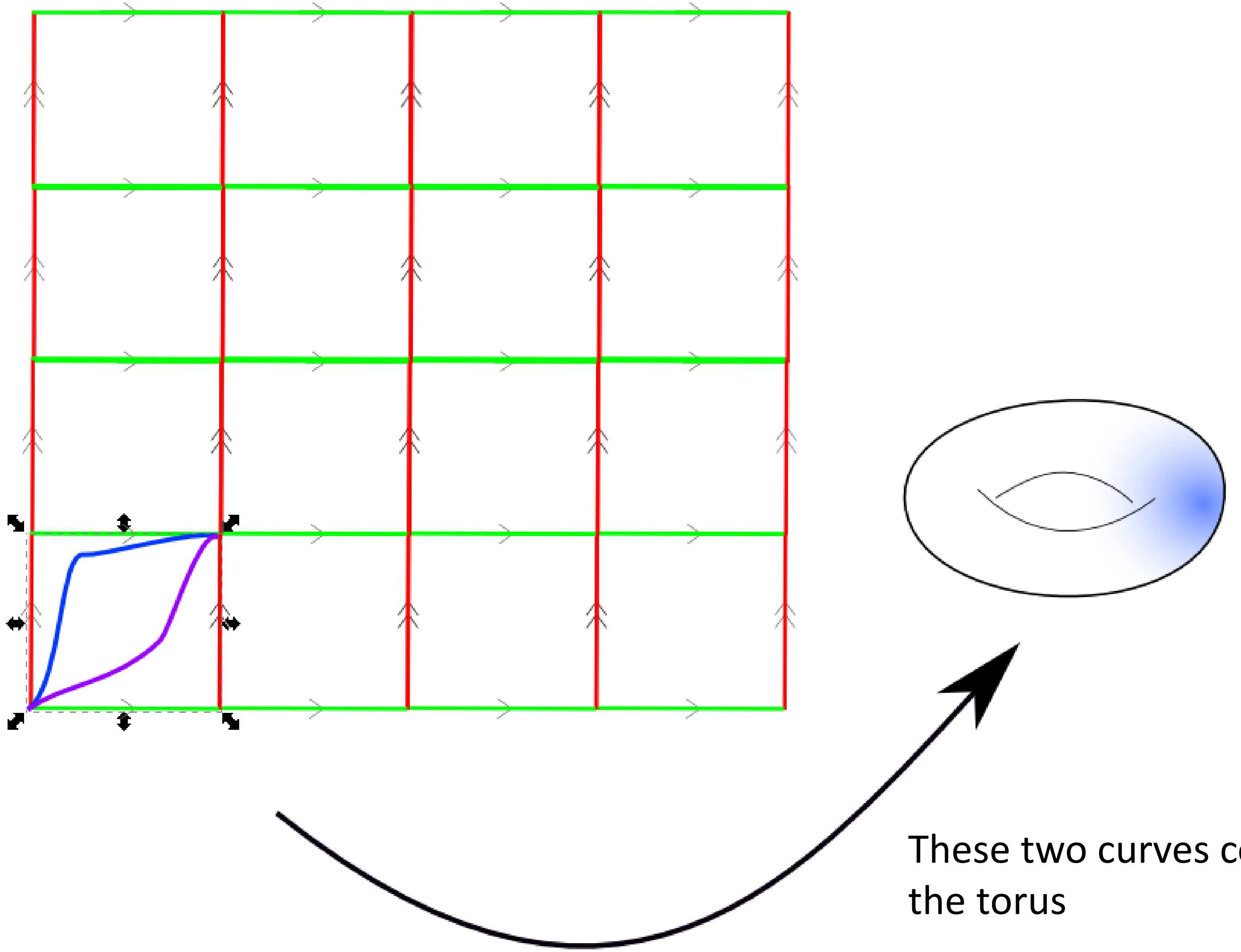

These two curves correspond to a homotopic closed curves on the torus

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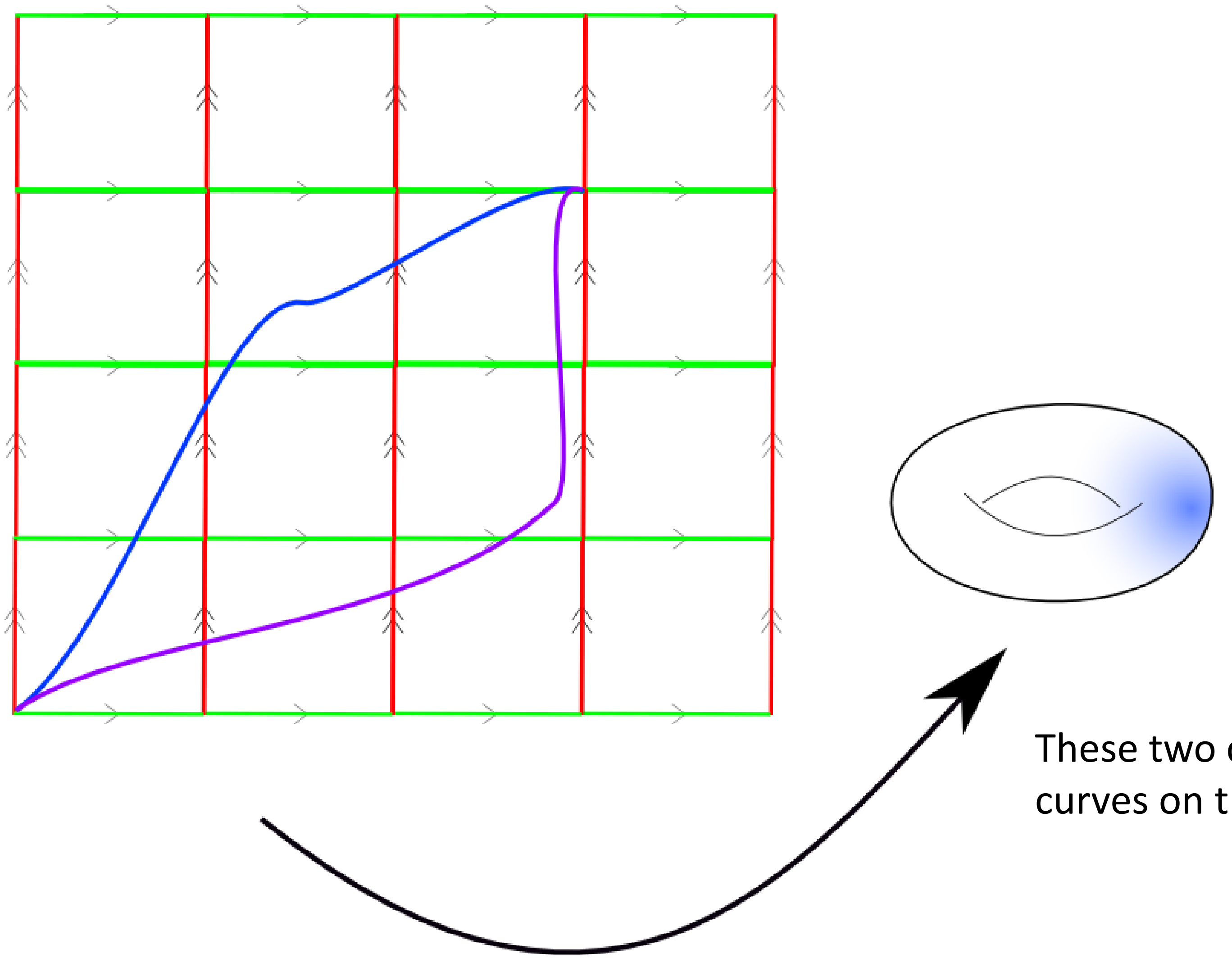

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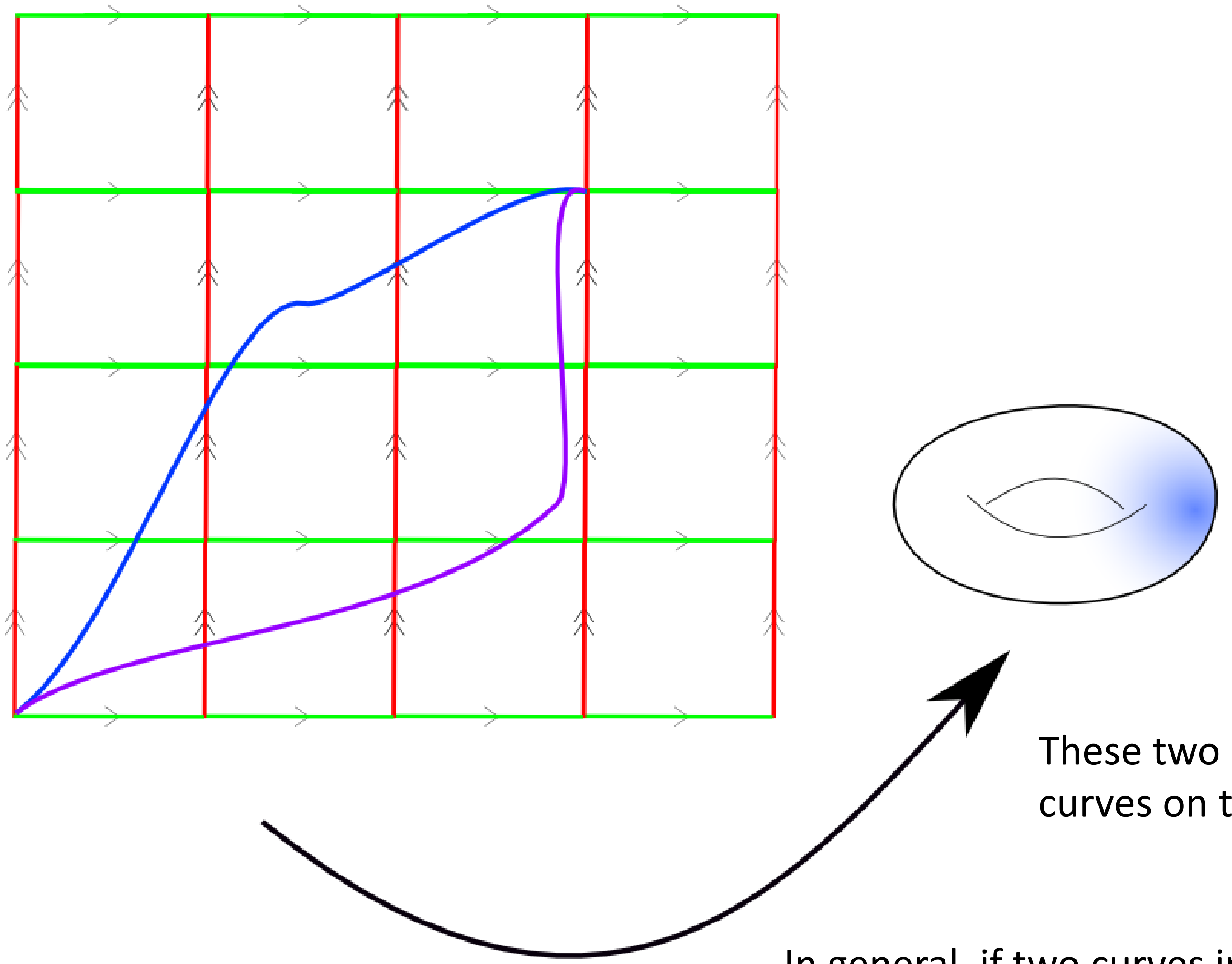

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In general, if two curves in the plane both of them start at (0,0) and end at (m,n) correspond to homotopic curves on the torus

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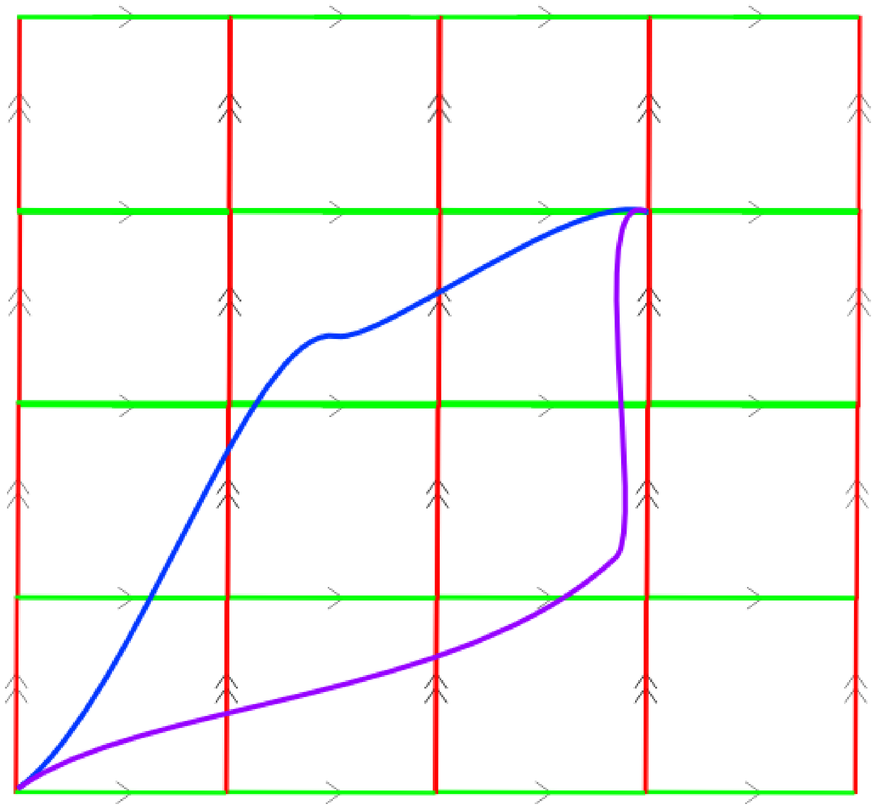

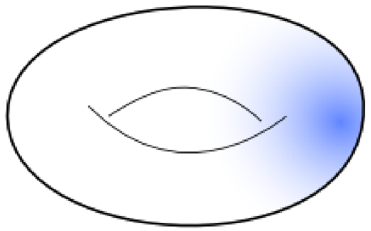

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In other words, the homotopy class of a closed curve on the torus is determined entirely by its starting and end points in the plane

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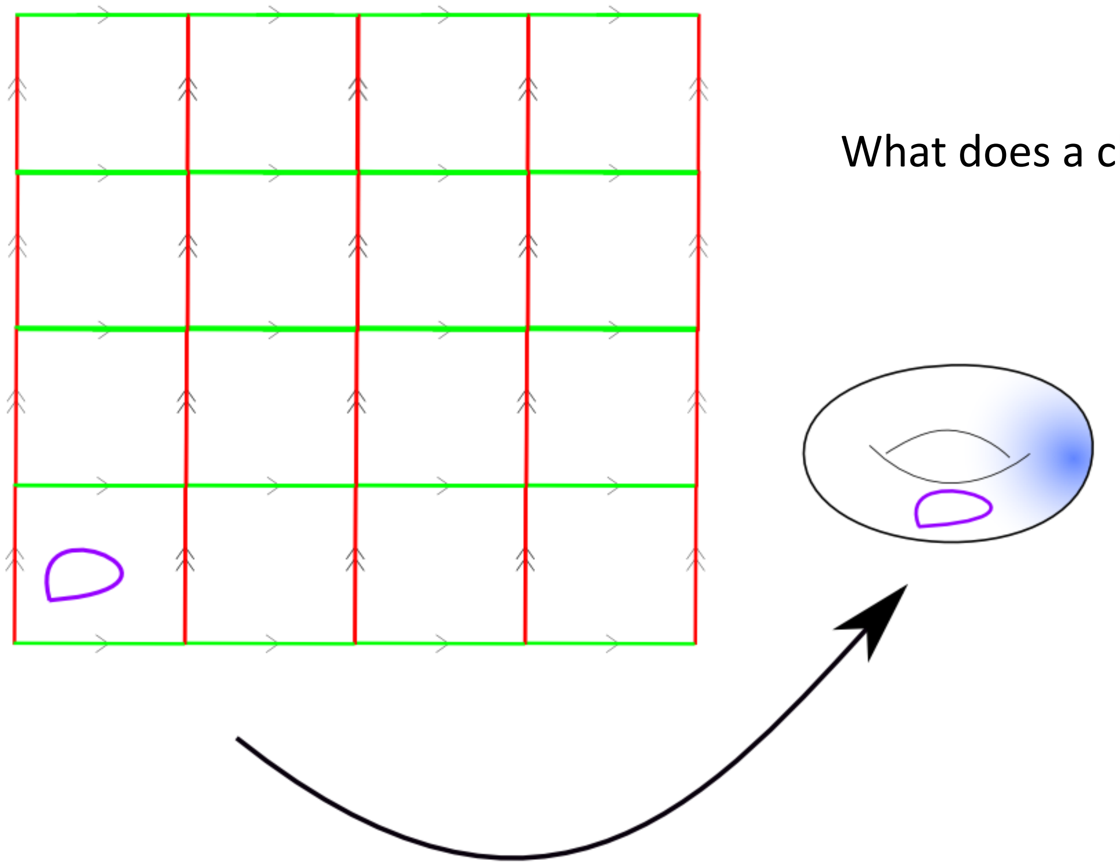
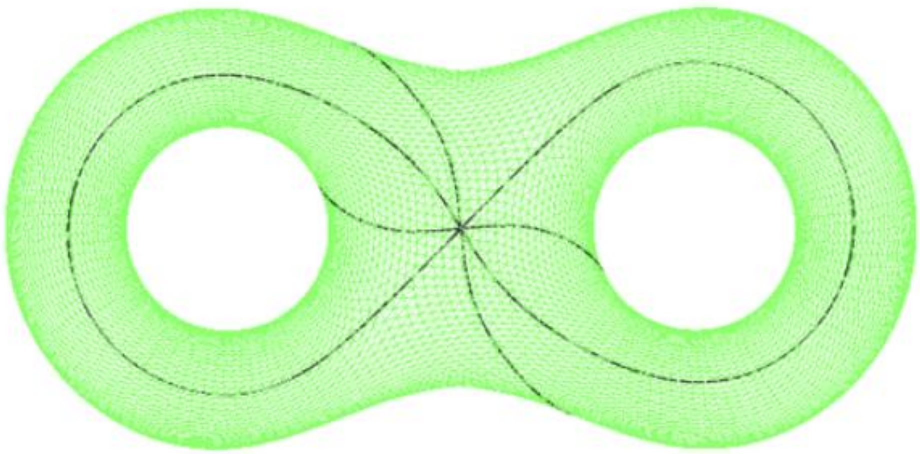


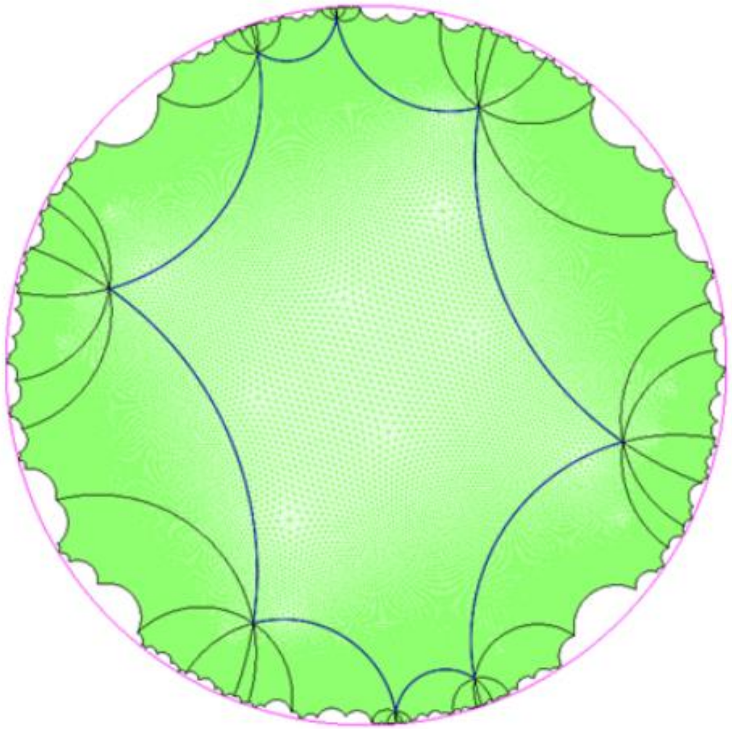
What does a closed curve in the plane correspond to?

covering space

The map between the plane and the torus is called a covering map. The plane is called a covering space. There are many coverings for the torus. The plane is special since it is simply connected. When a covering map is simply connected we say that the covering is universal.

Covering space for genus 2





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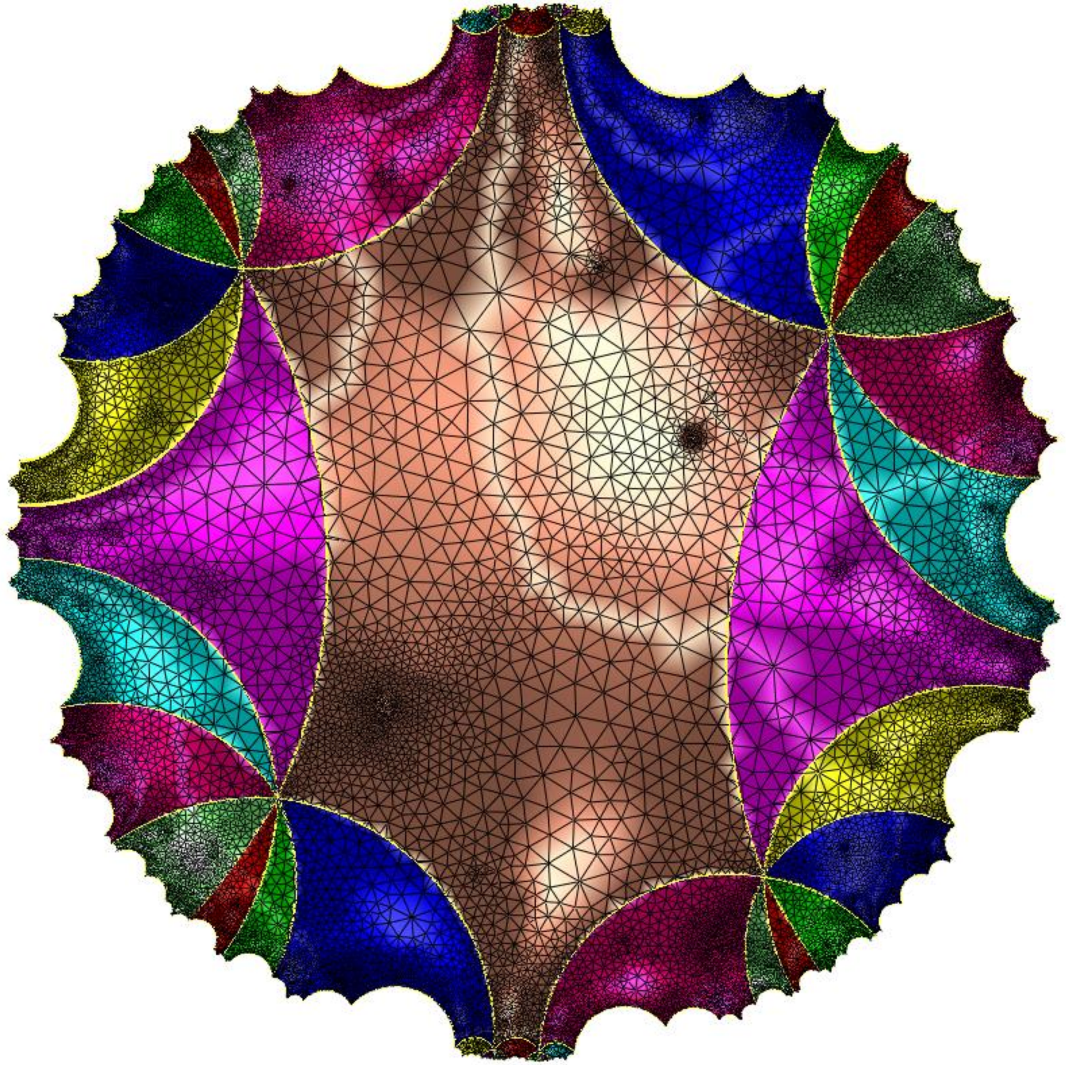

Covering space for genus 2

The universal covering space of orientable closed surfaces are the sphere (genus zero), the plane (genus one) and the disk (high genus).

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